

## Foreword

The twenty-fifth AMS Summer Research Institute was devoted to automorphic forms, representations and  $L$ -functions. It was held at Oregon State University, Corvallis, from July 11 to August 5, 1977, and was financed by a grant from the National Science Foundation. The Organizing Committee consisted of A. Borel, W. Casselman (cochairmen:), P. Deligne, H. Jacquet, R. P. Langlands, and J. Tate. The papers in this volume consist of the Notes of the Institute, mostly in revised form, and of a few papers written later.

A main goal of the Institute was the discussion of the  $L$ -functions attached to automorphic forms on, or automorphic representations of, reductive groups, the local and global problems pertaining to them, and of their relations with the  $L$ -functions of algebraic number theory and algebraic geometry, such as Artin  $L$ -functions and Hasse-Weil zeta functions. This broad topic, which goes back to E. Hecke, C. L. Siegel and others, has undergone in the last few years and is undergoing even now a considerable development, in part through the systematic use of infinite dimensional representations, in the framework of adelic groups. This development draws on techniques from several areas, some of rather difficult access. Therefore, besides seminars and lectures on recent and current work and open problems, the Institute also featured lectures (and even series of lectures) of a more introductory character, including background material on reductive groups, their representations, number theory, as well as an extensive treatment of some relatively simple cases.

The papers in this volume are divided into four main sections, reflecting to some extent the nature of the prerequisites. I is devoted to the structure of reductive groups and infinite dimensional representations of reductive groups over local fields. Five of the papers supply some basic background material, while the others are concerned with recent developments. II is concerned with automorphic forms and automorphic representations, with emphasis on the analytic theory. The first four papers discuss some basic facts and definitions pertaining to those, and the passage from one to the other. Two papers are devoted to Eisenstein series and the trace formula, first for  $GL_2$  and then in more general cases. In fact, the trace formula and orbital integrals turned out to be recurrent themes for the whole Institute and are featured in several papers in the other sections as well. The main theme of the last four papers is the restriction of the oscillator representation of the metaplectic group to dual reductive pairs of subgroups, first in general and then in more special cases.

III begins with the background material on number theory, chiefly on Weil groups and their  $L$ -functions. It then turns to the  $L$ -functions attached to automorphic representations, various ways to construct them, their (conjectured or proven) properties and local and global problems pertaining to them. The remaining papers are mostly devoted to the base change problem for  $GL_2$  and its applications to the proof of holomorphy of certain nonabelian Artin series.

Finally, IV relates automorphic representations and arithmetical algebraic geometry. Over function fields, it gives an introduction to the work of Drinfeld for

$GL_2$ , which constructs systems of  $l$ -adic representations whose  $L$ -series is a given automorphic  $L$ -function. Over number fields, it is mainly concerned with problems on Shimura varieties: canonical models, the point of their reductions modulo prime ideals, and Hasse-Weil zeta functions.

This Institute emphasized representations so that, at least formally, the primary object of concern was an automorphic representation rather than an automorphic form. However, there is no substantial difference between the two, and this should not hide the fact that the theory is a direct outgrowth of the classical theory of automorphic forms. In order to give a comprehensive treatment of our subject matter and yet not produce too heavy a schedule, it was decided to omit a number of topics on automorphic forms which do not fit well at present into the chosen framework. For example, the Institute was planned to have little overlap with the Conference on Modular Functions of One Variable held in Bonn (1976). The reader is referred to the Proceedings of the latter (Springer Lecture Notes 601, 627) and to those of its predecessor (Springer Lecture Notes 320, 350, 476) for some of those topics and a more classical point of view. Also, some topics of considerable interest in themselves such as reductive groups, their infinite dimensional representations, or moduli varieties, were discussed chiefly in function of the needs of the main themes of the Institute.

These Proceedings appear in two parts, the first one contains sections I and II, and the second one sections III and IV.

A. BOREL  
W. CASSELMAN