Introduction

The CRC Concise Encyclopedia of Mathematics is a compendium of mathematical definitions, formulas, figures, tabulations, and references. It is written in an informal style intended to make it accessible to a broad spectrum of readers with a wide range of mathematical backgrounds and interests. Although mathematics is a fascinating subject, it all too frequently is clothed in specialized jargon and dry formal exposition that make many interesting and useful mathematical results inaccessible to laypeople. This problem is often further compounded by the difficulty in locating concrete and easily understood examples. To give perspective to a subject, I find it helpful to learn why it is useful, how it is connected to other areas of mathematics and science, and how it is actually implemented. While a picture may be worth a thousand words, explicit examples are worth at least a few hundred! This work attempts to provide enough details to give the reader a flavor for a subject without getting lost in minutiae. While absolute rigor may suffer somewhat, I hope the improvement in usefulness and readability will more than make up for the deficiencies of this approach.

The format of this work is somewhere between a handbook, a dictionary, and an encyclopedia. It differs from existing dictionaries of mathematics in a number of important ways. First, the entire text and all the equations and figures are available in searchable electronic form on CD-ROM. Second, the entries are extensively cross-linked and cross-referenced, not only to related entries but also to many external sites on the Internet. This makes locating information very convenient. It also provides a highly efficient way to "navigate" from one related concept to another, a feature that is especially powerful in the electronic version. Standard mathematical references, combined with a few popular ones, are also given at the end of most entries to facilitate additional reading and exploration. In the interests of offering abundant examples, this work also contains a large number of explicit formulas and derivations, providing a ready place to locate a particular formula, as well as including the framework for understanding where it comes from.

The selection of topics in this work is more extensive than in most mathematical dictionaries (e.g., Borowski and Borwein's *HarperCollins Dictionary of Mathematics* and Jeans and Jeans' *Mathematics Dictionary*). At the same time, the descriptions are more accessible than in "technical" mathematical encyclopedias (e.g., Hazewinkel's *Encyclopaedia of Mathematics* and Iyanaga's *Encyclopedic Dictionary of Mathematics*). While the latter remain models of accuracy and rigor, they are not terribly useful to the undergraduate, research scientist, or recreational mathematician. In this work, the most useful, interesting, and entertaining (at least to my mind) aspects of topics are discussed in addition to their technical definitions. For example, in my entry for pi (π), the definition in terms of the diameter and circumference of a circle is supplemented by a great many formulas and series for pi, including some of the amazing discoveries of Ramanujan. These formulas are comprehensible to readers with only minimal mathematical background, and are interesting to both those with and without formal mathematics training. However, they have not previously been collected in a single convenient location. For this reason, I hope that, in addition to serving as a reference source, this work has some of the same flavor and appeal of Martin Gardner's delightful *Scientific American* columns.

Everything in this work has been compiled by me alone. I am an astronomer by training, but have picked up a fair bit of mathematics along the way. It never ceases to amaze me how mathematical connections weave their way through the physical sciences. It frequently transpires that some piece of recently acquired knowledge turns out to be just what I need to solve some apparently unrelated problem. I have therefore developed the habit of picking up and storing away odd bits of information for future use. This work has provided a mechanism for organizing what has turned out to be a fairly large collection of mathematics. I have also found it very difficult to find clear yet accessible explanations of technical mathematics unless I already have some familiarity with the subject. I hope this encyclopedia will provide jumping-off points for people who are interested in the subjects listed here but who, like me, are not necessarily experts.

The encyclopedia has been compiled over the last 11 years or so, beginning in my college years and continuing during graduate school. The initial document was written in *Microsoft Word*® on a Mac Plus® computer, and had reached about 200 pages by the time I started graduate school in 1990. When Andrew Treverrow made his OzTeX program available for the Mac, I began the task of converting all my documents to TeX, resulting in a vast improvement in readability. While undertaking the *Word* to TeX conversion, I also began cross-referencing entries, anticipating that eventually I would be able to convert the entire document to hypertext. This hope was realized beginning in 1995, when the Internet explosion was in full swing and I learned of Nikos Drakos's excellent TeX to HTML converter, LaTeX2HTML. After some additional effort, I was able to post an HTML version of my encyclopedia to the World Wide Web, currently located at `www.astro.virginia.edu/~eww6n/math/`.

The selection of topics included in this compendium is not based on any fixed set of criteria, but rather reflects my own random walk through mathematics. In truth, there is no good way of selecting topics in such a work. The mathematician James Sylvester may have summed up the situation most aptly. According to Sylvester (as quoted in the introduction to Ian Stewart's book *From Here to Infinity*), "Mathematics is not a book confined within a cover and bound between brazen clasps, whose contents it needs only patience to ransack; it is not a mine, whose treasures may take long to reduce into possession, but which fill only a limited number of veins and lodes; it is not a soil, whose fertility can be exhausted by the yield of successive harvests; it is not a continent or an ocean, whose area can be mapped out and its contour defined; it is as limitless as that space which it finds too narrow for its aspiration; its possibilities are as infinite as the worlds which are forever crowding in and multiplying upon the astronomer's gaze; it is as incapable of being restricted within assigned boundaries or being reduced to definitions of permanent validity, as the consciousness of life."

Several of Sylvester's points apply particularly to this undertaking. As he points out, mathematics itself cannot be confined to the pages of a book. The results of mathematics, however, are shared and passed on primarily through the printed (and now electronic) medium. While there is no danger of mathematical results being lost through lack of dissemination, many people miss out on fascinating and useful mathematical results simply because they are not aware of them. Not only does collecting many results in one place provide a single starting point for mathematical exploration, but it should also lessen the aggravation of encountering explanations for new concepts which themselves use unfamiliar terminology. In this work, the reader is only a cross-reference (or a mouse click) away from the necessary background material. As to Sylvester's second point, the very fact that the quantity of mathematics is so great means that any attempt to catalog it with any degree of completeness is doomed to failure. This certainly does not mean that it's not worth trying. Strangely, except for relatively small works usually on particular subjects, there do not appear to have been any substantial attempts to collect and display in a place of prominence the treasure trove of mathematical results that have been discovered (invented?) over the years (one notable exception being Sloane and Plouffe's *Encyclopedia of Integer Sequences*). This work, the product of the "gazing" of a single astronomer, attempts to fill that omission.

Finally, a few words about logistics. Because of the alphabetical listing of entries in the encyclopedia, neither table of contents nor index are included. In many cases, a particular entry of interest can be located from a cross-reference (indicated in SMALL CAPS TYPEFACE in the text) in a related article. In addition, most articles are followed by a "see also" list of related entries for quick navigation. This can be particularly useful if you are looking for a specific entry (say, "Zeno's Paradoxes"), but have forgotten the exact name. By examining the "see also" list at bottom of the entry for "Paradox," you will likely recognize Zeno's name and thus quickly locate the desired entry.

The alphabetization of entries contains a few peculiarities which need mentioning. All entries beginning with a numeral are ordered by increasing value and appear before the first entry for "A." In multiple-word entries containing a space or dash, the space or dash is treated as a character which precedes "a," so entries appear in the following order: "Sum," "Sum P...," "Sum-P...," and "Summary." One exception is that in a series of entries where a trailing "s" appears in some and not others, the trailing "s" is ignored in the alphabetization. Therefore, entries involving Euclid would be alphabetized as follows: "Euclid's Axioms," "Euclid Number," "Euclidean Algorithm." Because of the non-standard nomenclature that ensues from naming mathematical results after their discoverers, an important result such as the "Pythagorean Theorem" is written variously as "Pythagoras's Theorem," the "Pythagoras Theorem," etc. In this encyclopedia, I have endeavored to use the most widely accepted form. I have also tried to consistently give entry titles in the singular (e.g., "Knot" instead of "Knots").

In cases where the same word is applied in different contexts, the context is indicated in parentheses or appended to the end. Examples of the first type are "Crossing Number (Graph)" and "Crossing Number (Link)." Examples of the second type are "Convergent Sequence" and "Convergent Series." In the case of an entry like "Euler Theorem," which may describe one of three or four different formulas, I have taken the liberty of adding descriptive words ("Euler's *Something* Theorem") to all variations, or kept the standard name for the most commonly used variant and added descriptive words for the others. In cases where specific examples are derived from a general concept, em dashes (—) are used (for example, "Fourier Series," "Fourier Series—Power Series," "Fourier Series—Square Wave," "Fourier Series—Triangle"). The decision to put a possessive 's at the end of a name or to use a lone trailing apostrophe is based on whether the final "s" is pronounced. "Gauss's Theorem" is therefore written out, whereas "Archimedes' Recurrence Formula" is not. Finally, given the absence of a definitive stylistic convention, plurals of numerals are written without an apostrophe (e.g., 1990s instead of 1990's).

In an endeavor of this magnitude, errors and typographical mistakes are inevitable. The blame for these lies with me alone. Although the current length makes extensive additions in a printed version problematic, I plan to continue updating, correcting, and improving the work.

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