Contents

Introduction	Xi
CHAPTER 1 Investment Guarantees	1
Introduction	1
Major Benefit Types	4
Contract Types	5
Equity-Linked Insurance and Options	7
Provision for Equity-Linked Liabilities	11
Pricing and Capital Requirements	14
CHAPTER 2	15
Modeling Long-Term Stock Returns	10
Introduction	15
Deterministic or Stochastic?	15
Economical Theory or Statistical Method?	17
The Data	18
The Lognormal Model	24
Autoregressive Models	27
ARCH(1)	28
Regime-Switching Lognormal Model (RSLN)	30
The Empirical Model	36
The Stable Distribution Family	37
General Stochastic Volatility Models	38
The Wilkie Model	39
Vector Autoregression	45
CHAPTER 3	47
Maximum Likelihood Estimation for Stock Return Models	47
Introduction	47
Properties of Maximum Likelihood Estimators	49
Some Limitations of Maximum Likelihood Estimation	52

viii	CONTENTS

Using MLE for TSE and S&P Data	53
Likelihood-Based Model Selection	60
Moment Matching	63
AULDED 4	
CHAPTER 4 The Left-Tail Calibration Method	65
Introduction	65
Quantile Matching	66
The Canadian Calibration Table	67
Quantiles for Accumulation Factors: The Empirical Evidence	68
The Lognormal Model	70
Analytic Calibration of Other Models	72
Calibration by Simulation	75
CHAPTER 5	
Markov Chain Monte Carlo (MCMC) Estimation	77
Bayesian Statistics	77
Markov Chain Monte Carlo—An Introduction	79
The Metropolis-Hastings Algorithm (MHA)	81
MCMC for the RSLN Model	85
Simulating the Predictive Distribution	90
CHAPTER 6	
Modeling the Guarantee Liability	95
Introduction	95
The Stochastic Processes	96
Simulating the Stock Return Process	97
Notation	98
Guaranteed Minimum Maturity Benefit	100
Guaranteed Minimum Death Benefit	101
Example	101
Guaranteed Minimum Accumulation Benefit	102
GMAB Example	104
Stochastic Simulation of Liability Cash Flows	108
The Voluntary Reset	112
CHAPTER 7	
A Review of Option Pricing Theory	115
Introduction	115
The Guarantee Liability as a Derivative Security	116

Contents	Kİ.

Replication and No-Arbitrage Pricing	116
The Black-Scholes-Merton Assumptions	123
The Black-Scholes-Merton Results	124
The European Put Option	126
The European Call Option	128
Put-Call Parity	128
Dividends	129
Exotic Options	130
CHAPTER 8	400
Dynamic Hedging for Separate Account Guarantees	133
Introduction	133
Black-Scholes Formulae for Segregated Fund Guarantees	134
Pricing by Deduction from the Separate Account	142
The Unhedged Liability	143
Examples	151
CHAPTER 9	
Risk Measures	157
Introduction	157
The Quantile Risk Measure	159
The Conditional Tail Expectation Risk Measure	163
Quantile and CTE Measures Compared	167
Risk Measures for GMAB Liability	169
Risk Measures for VA Death Benefits	173
CHAPTER 10	
Emerging Cost Analysis	177
Decisions	177
Capital Requirements: Actuarial Risk Management	180
Capital Requirements: Dynamic-Hedging Risk Management	184
Emerging Costs with Solvency Capital	188
Example: Emerging Costs for 20-Year GMAB	189
CHAPTER 11	
Forecast Uncertainty	195
Sources of Uncertainty	195
Random Sampling Error	196
Variance Reduction	201
Parameter Uncertainty	213
Model Uncertainty	219

X CONTENTS

CHAPTER 12 Guaranteed Annuity Options	221
Introduction	221
Interest Rate and Annuity Modeling Actuarial Modeling	224 228
Dynamic Hedging	230
Static Replication	235
CHAPTER 13	
Equity-Indexed Annuities	237
Introduction	237
Contract Design	239
Valuing the Embedded Options	243
PTP Option Valuation	244
Compound Annual Ratchet Valuation	247
The Simple Annual Ratchet Option Valuation	257 258
The High Water Mark Option Valuation Dynamic Hedging for the PTP Option	260
Conclusions and Further Reading	263
APPENDIX A Mortality and Survival Probabilities	265
APPENDIX B The GMAB Option Price	271
APPENDIX C Actuarial Notation	273
REFERENCES	275
INDEX	281