

Contents

1	Introduction	13
I	Formalisms for Computation: Register Machines, Exponential Diophantine Equations, & Pure LISP	19
2	Register Machines	23
2.1	Introduction	23
2.2	Pascal's Triangle Mod 2	26
2.3	LISP Register Machines	30
2.4	Variables Used in Arithmetization	45
2.5	An Example of Arithmetization	49
2.6	A Complete Example of Arithmetization	58
2.7	Expansion of \Rightarrow 's	63
2.8	Left-Hand Side	71
2.9	Right-Hand Side	75
3	A Version of Pure LISP	79
3.1	Introduction	79
3.2	Definition of LISP	81
3.3	Examples	89
3.4	LISP in LISP I	93
3.5	LISP in LISP II	94
3.6	LISP in LISP III	98

4	The LISP Interpreter EVAL	103
4.1	Register Machine Pseudo-Instructions	103
4.2	EVAL in Register Machine Language	106
4.3	The Arithmetization of EVAL	123
4.4	Start of Left-Hand Side	129
4.5	End of Right-Hand Side	131
 II Program Size, Halting Probabilities, Randomness, & Metamathematics		135
5	Conceptual Development	139
5.1	Complexity via LISP Expressions	139
5.2	Complexity via Binary Programs	145
5.3	Self-Delimiting Binary Programs	146
5.4	Omega in LISP	148
6	Program Size	157
6.1	Introduction	157
6.2	Definitions	158
6.3	Basic Identities	162
6.4	Random Strings	174
7	Randomness	179
7.1	Introduction	179
7.2	Random Reals	184
8	Incompleteness	197
8.1	Lower Bounds on Information Content	197
8.2	Random Reals: First Approach	200
8.3	Random Reals: $ Axioms $	202
8.4	Random Reals: $H(Axioms)$	209
9	Conclusion	213
10	Bibliography	215
A	Implementation Notes	221

<i>CONTENTS</i>	9
B S-expressions of Size N	223
C Back Cover	233