Contents

1 Recurrent Problems 1

1.1 The Tower of Hanoi 1
1.2 Lines in the Plane 4
1.3 The Josephus Problem 8
Exercises 17

2 Sums 21

2.1 Notation 21
2.2 Sums and Recurrences 25
2.3 Manipulation of Sums 30
2.4 Multiple Sums 34
2.5 General Methods 41
2.6 Finite and Infinite Calculus 47
2.7 Infinite Sums 56
Exercises 62

3 Integer Functions 67

3.1 Floors and Ceilings 67
3.2 Floor/Ceiling Applications 70
3.3 Floor/Ceiling Recurrences 78
3.4 'mod': The Binary Operation 81
3.5 Floor/Ceiling Sums 86
Exercises 95

4 Number Theory 102

4.1 Divisibility **102**
4.2 Primes 105
4.3 Prime Examples 107
4.4 Factorial Factors 111
4.5 Relative Primality 115
4.6 'mod': The Congruence Relation 123
4.7 Independent Residues 126
4.8 Additional Applications 129
4.9 Phi and Mu 133
Exercises 144

5 Binomial Coefficients 153

5.1 Basic Identities 153
5.2 Basic Practice 172

5.3 Tricks of the Trade 186
5.4 Generating Functions 196
5.5 Hypergeometric Functions 204
5.6 Hypergeometric Transformations 216
5.7 Partial Hypergeometric Sums 223
Exercises **230**

6 Special Numbers 243

6.1 Stirling Numbers 243
6.2 Eulerian Numbers 253
6.3 Harmonic Numbers 258
6.4 Harmonic Summation 265
6.5 Bernoulli Numbers 269
6.6 Fibonacci Numbers 276
6.7 Continuants 287
Exercises 295

7 Generating Functions 306

7.1 Domino Theory and Change 306
7.2 Basic Maneuvers 317
7.3 Solving Recurrences 323
7.4 Special Generating Functions 336
7.5 Convolutions 339
7.6 Exponential Generating Functions 350
7.7 Dirichlet Generating Functions **356**
Exercises **357**

8 Discrete Probability **367**

8.1 Definitions **367**
8.2 Mean and Variance **373**
8.3 Probability Generating Functions **380**
8.4 Flipping Coins **387**
8.5 Hashing 397
Exercises 413

9 Asymptotics **425**

9.1 A Hierarchy **426**
9.2 0 Notation 429
9.3 0 Manipulation **436**
9.4 Two Asymptotic Tricks 449
9.5 Euler's Summation Formula **455**
9.6 Final Summations **462**
Exercises **475**

A Answers to Exercises **483**

B Bibliography **578**

C Credits for Exercises **601**

Index **606**

List of Tables **624**