

Contents

| | | |
|----------|--|-----------|
| 1 | Linear spaces; normed spaces; first examples | 9 |
| 1.1 | Linear spaces | 9 |
| 1.2 | Normed spaces; first examples | 11 |
| 1.2.1 | Hölder inequality. | 12 |
| 1.2.2 | Minkowski inequality | 13 |
| 1.3 | Completeness; completion | 16 |
| 1.3.1 | Construction of completion | 17 |
| 1.4 | Exercises | 18 |
| 2 | Hilbert spaces | 21 |
| 2.1 | Basic notions; first examples | 21 |
| 2.1.1 | Cauchy-Schwartz inequality | 22 |
| 2.1.2 | Bessel's inequality | 23 |
| 2.1.3 | Gram-Schmidt orthogonalization procedure | 24 |
| 2.1.4 | Parseval's equality | 25 |
| 2.2 | Projections; decompositions | 27 |
| 2.2.1 | Separable case | 27 |
| 2.2.2 | Uniqueness of the distance from a point to a convex set: the geometric meaning | 27 |
| 2.2.3 | Orthogonal decomposition | 28 |
| 2.3 | Linear functionals | 29 |
| 2.3.1 | Linear functionals in a general linear space | 29 |
| 2.3.2 | Bounded linear functionals in normed spaces. The norm of a functional | 31 |
| 2.3.3 | Bounded linear functionals in a Hilbert space | 32 |
| 2.3.4 | An Example of a non-separable Hilbert space: | 32 |
| 2.4 | Exercises | 33 |

| | | |
|----------|---|-----------|
| 3 | The dual space X^* | 39 |
| 3.1 | Hahn-Banach theorem and its first consequences | 39 |
| 3.2 | Dual Spaces | 41 |
| 3.3 | Exercises: | 42 |
| 4 | Bounded linear operators | 43 |
| 4.1 | Completeness of the space of bounded linear operators | 43 |
| 4.2 | Examples of linear operators | 44 |
| 4.3 | Compact operators | 45 |
| 4.3.1 | Compact sets | 46 |
| 4.3.2 | The space of compact operators | 48 |
| 4.4 | Dual Operators | 48 |
| 4.5 | Different convergences in the space $L(X)$ of bounded operators | 50 |
| 4.6 | Invertible Operators | 52 |
| 4.7 | Exercises | 52 |
| 5 | Spectral theory | 57 |
| 5.1 | Classification of spectrum | 57 |
| 5.2 | Fredholm Theory of compact operators | 58 |
| 5.3 | Exercises | 63 |
| 6 | Self adjoint compact operators | 65 |
| 6.1 | General Properties | 65 |
| 6.2 | Exercises | 72 |
| 7 | Self-adjoint bounded operators | 73 |
| 7.1 | Order in the space of symmetric operators | 73 |
| 7.1.1 | Properties | 73 |
| 7.2 | Projections (projection operators) | 77 |
| 7.2.1 | Some properties of projections in linear spaces | 77 |
| 8 | Functions of operators | 79 |
| 8.1 | Properties of this correspondence ($\varphi_i \in K$) | 80 |
| 8.2 | The main inequality | 82 |
| 8.3 | Simple spectrum | 85 |
| 9 | Spectral theory of unitary operators | 87 |
| 9.1 | Spectral properties | 87 |

| | |
|--|------------|
| 10 The Fundamental Theorems. | 91 |
| 10.1 The open mapping theorem | 92 |
| 10.2 The Closed Graph Theorem | 94 |
| 10.3 The Banach-Steinhaus Theorem | 95 |
| 10.4 Bases In Banach Spaces | 99 |
| 10.5 Hahn-Banach Theorem. Linear functionals | 100 |
| 10.6 Extremal points; The Krein-Milman Theorem | 108 |
| 11 Banach algebras | 111 |
| 11.1 Analytic functions | 114 |
| 11.2 Radicals | 118 |
| 11.3 Involutions | 120 |
| 12 Unbounded self-adjoint and symmetric operators in H | 127 |
| 12.1 More Properties Of Operators | 131 |
| 12.2 The Spectrum $\sigma(A)$ | 132 |
| 12.3 Elements Of The “Graph Method” | 133 |
| 12.4 Reduction Of Operator | 134 |
| 12.5 Cayley Transform | 136 |