Preface

These notes are intended to be of use to Third year Electrical and Electronic Engineers at the University of Western Australia coming to grips with Complex Function Theory.

There are many text books for just this purpose, and I have insufficient time to write a text book, so this is not a substitute for, say, Matthews and Howell's *Complex Analysis for Mathematics and Engineering*,[1], but perhaps a complement to it. At the same time, knowing how reluctant students are to use a textbook (except as a talisman to ward off evil) I have tried to make these notes sufficient, in that a student who reads them, understands them, and does the exercises in them, will be able to use the concepts and techniques in later years. It will also get the student comfortably through the examination. The shortness of the course, 20 lectures, for covering Complex Analysis, either presupposes genius (90% perspiration) on the part of the students or material skipped. These notes are intended to fill in some of the gaps that will inevitably occur in lectures. It is a source of some disappointment to me that I can cover so little of what is a beautiful subject, rich in applications and connections with other areas of mathematics. This is, then, a sort of sampler, and only touches the elements.

Styles of Mathematical presentation change over the years, and what was deemed acceptable rigour by Euler and Gauss fails to keep modern purists content. McLachlan, [2], clearly smarted under the criticisms of his presentation, and he goes to some trouble to explain in later editions that the book is intended for a different audience from the purists who damned him. My experience leads me to feel that the need for rigour has been developed to the point where the intuitive and geometric has been stunted. Both have a part in mathematics, which grows out of the conflict between them. But it seems to me more important to penetrate to the ideas in a sloppy, scruffy but serviceable way, than to reduce a subject to predicate calculus and omit the whole reason for studying it. There is no known means of persuading a hardheaded engineer that a subject merits his time and energy when it has been turned into an elaborate game. He, or increasingly she, wants to see two elements at an early stage: procedures for solving problems which make a difference and concepts which organise the procedures into something intelligible. Carried to excess this leads to avoidance of abstraction and consequent loss of power later; there is a good reason for the purist's desire for rigour. But it asks too much of a third year student to focus on the underlying logic and omit the geometry.

I have deliberately erred in the opposite direction. It is easy enough for the student with a taste for rigour to clarify the ideas by consulting other books, and to wind up as a logician if that is his choice. But it is hard to find in the literature any explicit commitment to getting the student to draw lots of pictures. It used to be taken for granted that a student would do that sort of thing, but now that the school syllabus has had Euclid expunged, the undergraduates cannot be expected to see drawing pictures or visualising surfaces as a natural prelude to calculation. There is a school of thought which considers geometric visualisation as immoral; and another which sanctions it only if done in private (and wash your hands before and afterwards). To my mind this imposes sterility, and constitutes an attempt by the bureaucrat to strangle the artist. [1] While I do not want to impose my informal images on anybody, if no mention is made of informal, intuitive ideas, many students never realise that there are any. All the good mathematicians I know have a rich supply of informal models which they use to think about mathematics, and it were as well to show students how this may be done. Since this seems to be the respect in which most of the text books are weakest, I have perhaps gone too far in the other direction, but then, I do not offer this as a text book. More of an antidote to some of the others.

I have talked to Electrical Engineers about Mathematics teaching, and they are strikingly consistent in what they want. Prior to talking to them, I feared that I'd find Engineers saying things like 'Don't bother with the ideas, forget about the pictures, just train them to do the sums'. There are, alas, Mathematicians who are convinced that this is how Engineers see the world, and I had supposed that there might be something in this belief. Silly me. In fact, it is simply quite wrong.

The Engineers I spoke to want Mathematicians to get across the abstract ideas in terms the students can grasp and use, so that the Engineers can subsequently rely on the student having those ideas as part of his or her

[1] The bureaucratic temper is attracted to mathematics while still at school, because it appears to be all about following rules, something the bureaucrat cherishes as the solution to the problems of life. Human beings on the other hand find this sufficiently repellant to be put off mathematics permanently, which is one of the ironies of education. My own attitude to the bureaucratic temper is rather that of Dave Allen's feelings about politicians. He has a soft spot for them. It's a bog in the West of Ireland.

thinking. Above all, they want the students to have clear pictures in their heads of what is happening in the mathematics. Since this is exactly what any competent Mathematician also wants his students to have, I haven't felt any need to change my usual style of presentation. This is informal and user-friendly as far as possible, with (because I am a Topologist by training and work with Engineers by choice) a strong geometric flavour.

I introduce Complex Numbers in a way which was new to me; I point out that a certain subspace of 2×2 matrices can be identifed with the plane $\mathbb{R}^2$, thus giving a simple rule for multiplying two points in $\mathbb{R}^2$: turn them into matrices, multiply the matrices, then turn the answer back into a point. I do it this way because (a) it demystifies the business of imaginary numbers, (b) it gives the Cauchy-Riemann conditions in a conceptually transparent manner, and (c) it emphasises that multiplication by a complex number is a similarity together with a rotation, a matter which is at the heart of much of the applicability of the complex number system. There are a few other advantages of this approach, as will be seen later on. After I had done it this way, Malcolm Hood pointed out to me that Copson, [3], had taken the same approach.[2]

Engineering students lead a fairly busy life in general, and the Sparkies have a particularly demanding load. They are also very practical, rightly so, and impatient of anything which they suspect is academic window-dressing. So far, I am with them all the way. They are, however, the main source of the belief among some mathematicians that peddling recipes is the only way to teach them. They do not feel comfortable with abstractions. Their goal tends to be examination passing. So there is some basic opposition between the students and me: I want them to be able to use the material in later years, they want to memorise the minimum required to pass the exam (and then forget it).

I exaggerate of course. For reasons owing to geography and history, this University is particularly fortunate in the quality of its students, and most of them respond well to the discovery that Mathematics makes sense. I hope that these notes will turn out to be enjoyable as well as useful, at least in retrospect.

But be warned:

[2]I am most grateful to Malcolm for running an editorial eye over these notes, but even more grateful for being a model of sanity and decency in a world that sometimes seems bereft of both.