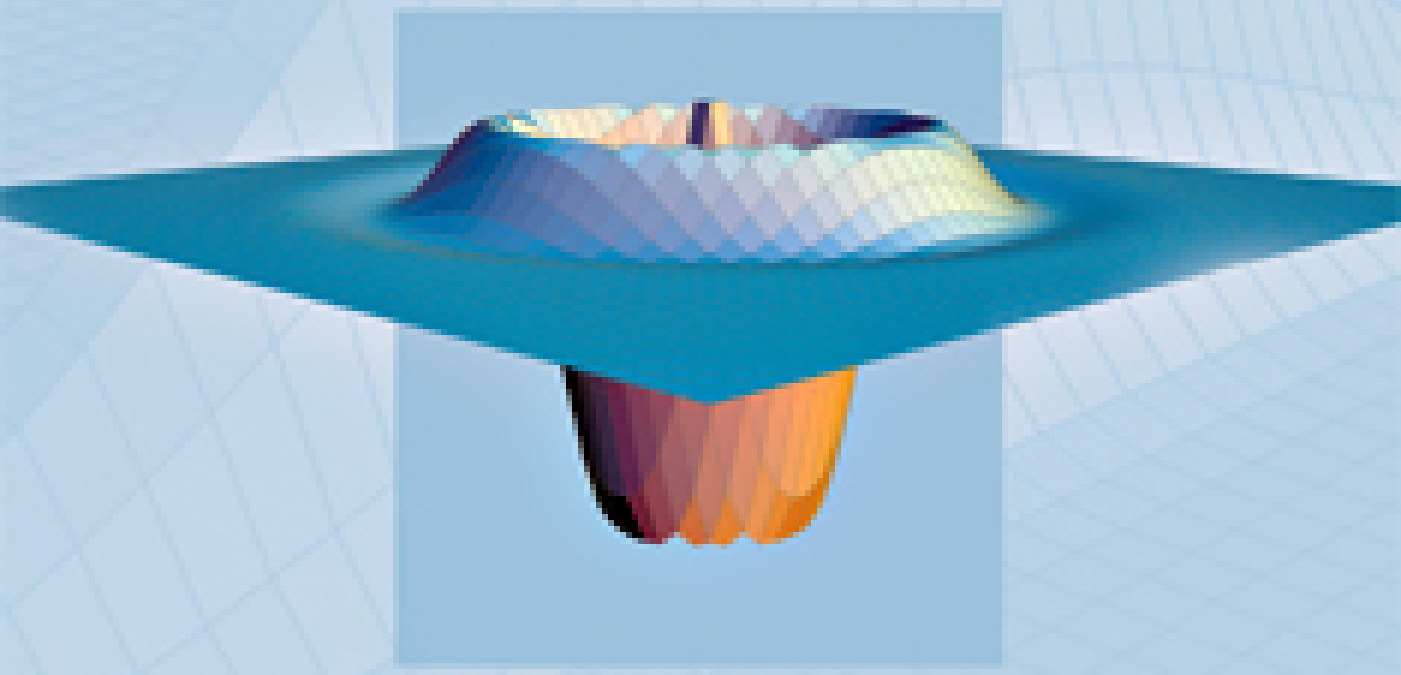
QUANTUM MECHANICS



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Quantum Mechanics

The important changes quantum mechanics has undergone in recent years are reflected in this new approach for students.

A strong narrative and over 300 worked problems lead the student from experiment, through general principles of the theory, to modern applications. Stepping through results allows students to gain a thorough understanding. Starting with basic quantum mechanics, the book moves on to more advanced theory, followed by applications, perturbation methods and special fields, and ending with new developments in the field. Historical, mathematical, and philosophical boxes guide the student through the theory. Unique to this textbook are chapters on measurement and quantum optics, both at the forefront of current research. Advanced undergraduate and graduate students will benefit from this new perspective on the fundamental physical paradigm and its applications.

Online resources including solutions to selected problems and 200 figures, with color versions of some figures, are available at www.cambridge.org/Auletta.

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