

Contents

<i>List of figures</i>	page xi
<i>List of tables</i>	xvii
<i>List of definitions, principles, etc.</i>	xviii
<i>List of boxes</i>	xx
<i>List of symbols</i>	xxi
<i>List of abbreviations</i>	xxxii

Introduction	1
---------------------	----------

Part I Basic features of quantum mechanics

1 From classical mechanics to quantum mechanics	7
1.1 Review of the foundations of classical mechanics	7
1.2 An interferometry experiment and its consequences	12
1.3 State as vector	20
1.4 Quantum probability	28
1.5 The historical need of a new mechanics	31
Summary	40
Problems	41
Further reading	42
2 Quantum observables and states	43
2.1 Basic features of quantum observables	43
2.2 Wave function and basic observables	68
2.3 Uncertainty relation	82
2.4 Quantum algebra and quantum logic	92
Summary	96
Problems	97
Further reading	99
3 Quantum dynamics	100
3.1 The Schrödinger equation	101
3.2 Properties of the Schrödinger equation	107
3.3 Schrödinger equation and Galilei transformations	111
3.4 One-dimensional free particle in a box	113
3.5 Unitary transformations	117

3.6	Different pictures	125
3.7	Time derivatives and the Ehrenfest theorem	129
3.8	Energy–time uncertainty relation	130
3.9	Towards a time operator	135
	Summary	138
	Problems	139
	Further reading	140
4	Examples of quantum dynamics	141
4.1	Finite potential wells	141
4.2	Potential barrier	145
4.3	Tunneling	150
4.4	Harmonic oscillator	154
4.5	Quantum particles in simple fields	165
	Summary	169
	Problems	170
5	Density matrix	174
5.1	Basic formalism	174
5.2	Expectation values and measurement outcomes	177
5.3	Time evolution and density matrix	179
5.4	Statistical properties of quantum mechanics	180
5.5	Compound systems	181
5.6	Pure- and mixed-state representation	187
	Summary	188
	Problems	189
	Further reading	190
Part II More advanced topics		
6	Angular momentum and spin	193
6.1	Orbital angular momentum	193
6.2	Special examples	207
6.3	Spin	217
6.4	Composition of angular momenta and total angular momentum	226
6.5	Angular momentum and angle	239
	Summary	241
	Problems	242
	Further reading	244
7	Identical particles	245
7.1	Statistics and quantum mechanics	245
7.2	Wave function and symmetry	247
7.3	Spin and statistics	249

7.4	Exchange interaction	254
7.5	Two recent applications	255
	Summary	257
	Problems	257
	Further reading	258
8	Symmetries and conservation laws	259
8.1	Quantum transformations and symmetries	259
8.2	Continuous symmetries	264
8.3	Discrete symmetries	266
8.4	A brief introduction to group theory	267
	Summary	275
	Problems	275
	Further reading	276
9	The measurement problem in quantum mechanics	277
9.1	Statement of the problem	278
9.2	A brief history of the problem	284
9.3	Schrödinger cats	291
9.4	Decoherence	297
9.5	Reversibility/irreversibility	308
9.6	Interaction-free measurement	315
9.7	Delayed-choice experiments	320
9.8	Quantum Zeno effect	322
9.9	Conditional measurements or postselection	325
9.10	Positive operator valued measure	327
9.11	Quantum non-demolition measurements	335
9.12	Decision and estimation theory	341
	Summary	349
	Problems	351
	Further reading	353

Part III Matter and light

10	Perturbations and approximation methods	357
10.1	Stationary perturbation theory	357
10.2	Time-dependent perturbation theory	366
10.3	Adiabatic theorem	369
10.4	The variational method	371
10.5	Classical limit	372
10.6	Semiclassical limit and WKB approximation	378
10.7	Scattering theory	384
10.8	Path integrals	389
	Summary	398

Problems	399
Further reading	399
11 Hydrogen and helium atoms	401
11.1 Introduction	401
11.2 Quantum theory of the hydrogen atom	403
11.3 Atom and magnetic field	413
11.4 Relativistic corrections	423
11.5 Helium atom	426
11.6 Many-electron effects	431
Summary	436
Problems	437
Further reading	438
12 Hydrogen molecular ion	439
12.1 The molecular problem	439
12.2 Born–Oppenheimer approximation	440
12.3 Vibrational and rotational degrees of freedom	443
12.4 The Morse potential	447
12.5 Chemical bonds and further approximations	449
Summary	453
Problems	453
Further reading	454
13 Quantum optics	455
13.1 Quantization of the electromagnetic field	457
13.2 Thermodynamic equilibrium of the radiation field	462
13.3 Phase–number uncertainty relation	463
13.4 Special states of the electromagnetic field	465
13.5 Quasi-probability distributions	474
13.6 Quantum-optical coherence	481
13.7 Atom–field interaction	484
13.8 Geometric phase	497
13.9 The Casimir effect	501
Summary	506
Problems	507
Further reading	509
Part IV Quantum information: state and correlations	
14 Quantum theory of open systems	513
14.1 General considerations	514
14.2 The master equation	516

14.3	A formal generalization	523
14.4	Quantum jumps and quantum trajectories	528
14.5	Quantum optics and Schrödinger cats	533
	Summary	540
	Problems	541
	Further reading	542
15	State measurement in quantum mechanics	544
15.1	Protective measurement of the state	544
15.2	Quantum cloning and unitarity violation	548
15.3	Measurement and reversibility	550
15.4	Quantum state reconstruction	554
15.5	The nature of quantum states	564
	Summary	565
	Problems	565
	Further reading	566
16	Entanglement: non-separability	567
16.1	EPR	568
16.2	Bohm's version of the EPR state	573
16.3	HV theories	577
16.4	Bell's contribution	582
16.5	Experimental tests	595
16.6	Bell inequalities with homodyne detection	605
16.7	Bell theorem without inequalities	613
16.8	What is quantum non-locality?	619
16.9	Further developments about inequalities	623
16.10	Conclusion	625
	Summary	625
	Problems	626
	Further reading	627
17	Entanglement: quantum information and computation	628
17.1	Information and entropy	628
17.2	Entanglement and information	631
17.3	Measurement and information	639
17.4	Qubits	642
17.5	Teleportation	643
17.6	Quantum cryptography	646
17.7	Elements of quantum computation	650
17.8	Quantum algorithms and error correction	659
	Summary	671

Problems	672
Further reading	673
<i>Bibliography</i>	674
<i>Author index</i>	710
<i>Subject index</i>	716

Figures

1.1	Graphical representation of the Liouville theorem	<i>page</i> 12
1.2	Photoelectric effect	13
1.3	Mach–Zender interferometer	15
1.4	The Michelson–Morley interferometer	16
1.5	Interferometer for detecting gravitational waves	17
1.6	Interference in the Mach–Zender interferometer	17
1.7	Results of the experiment performed by Grangier, Roger, and Aspect	18
1.8	Oscillation of electric and magnetic fields	20
1.9	Polarization of classical light	21
1.10	Decomposition of an arbitrary vector $ a\rangle$	22
1.11	Poincaré sphere representation of states	28
1.12	Mach–Zender interferometer with the lower path blocked by the screen S	30
1.13	Black-body radiation intensity corresponding to the formula of Rayleigh–Jeans (1), Planck (2), and Wien (3)	32
1.14	Planck’s radiation curves in logarithmic scale for the temperatures of liquid nitrogen, melting ice, boiling water, melting aluminium, and the solar surface	33
1.15	Compton effect	34
1.16	Dulong–Petit’s, Einstein’s, and Debye’s predictions for specific heat	36
1.17	Lyman series for ionized helium	37
1.18	The Stern–Gerlach Experiment	39
1.19	Momentum conservation in the Compton effect	41
2.1	Polarization beam splitter	44
2.2	Change of basis	52
2.3	Filters	62
2.4	Two sequences of two rotations of a book	65
2.5	Probability distributions of position and momentum for a momentum eigenfunction	85
2.6	Probability distributions of position and momentum for a position eigenfunction	86
2.7	Time evolution of a classical degree of freedom in phase space and graphical representation of the uncertainty relation	88
2.8	Inverse proportionality between momentum and position uncertainties	89
2.9	Smooth complementarity between wave and particle	90
2.10	Illustration of the distributive law	93
2.11	Proposed interferometry and resulting non-Boolean algebra and Boolean subalgebras	94

2.12	Hasse diagrams of several Boolean and non-Boolean algebras	95
3.1	Positive potential vanishing at infinity	109
3.2	Potential function tending to finite values as $x \rightarrow \pm\infty$	110
3.3	Potential well	111
3.4	Relation between two different inertial reference frames \mathcal{R} and \mathcal{R}' under Galilei transformations	112
3.5	Particle in a box of dimension a	113
3.6	Energy levels of a particle in a one-dimensional box	115
3.7	First three energy eigenfunctions for a one-dimensional particle confined in a box of dimension a	115
3.8	Beam Splitters as unitary operators	119
3.9	Projector as a residue of the closed contour in a complex plane	124
3.10	A graphical representation of the apparatus proposed by Bohr	133
4.1	Schematic and asymmetric one-dimensional potential wells	142
4.2	Solution of Eq. (4.8)	143
4.3	Wave functions and probability densities for the first three eigenfunctions for the symmetric finite-well potential	144
4.4	Stepwise continuity	145
4.5	Potential barrier	147
4.6	Closed surface used to compute the flux of \mathbf{J}	148
4.7	Delta potential barrier	149
4.8	Classical turning points and quantum tunneling	151
4.9	Tunneling of α -particles	152
4.10	Carbon atoms shown by scanning tunneling microscopy	153
4.11	Potential and energy levels of the harmonic oscillator	154
4.12	Eigenfunctions for the one-dimensional harmonic oscillator	162
4.13	Potential energy corresponding to a particle in a uniform field	165
4.14	Triangular well	167
4.15	A quantum particle with energy E encounters a potential step of height $V_0 < E$	171
4.16	Rectangular potential barrier with finite width a	171
5.1	Representation of pure and mixed states on a sphere	187
6.1	Angular momentum of a classical particle	194
6.2	Levi-Civita tensor	195
6.3	Relationship between rectangular and spherical coordinates	200
6.4	s - and p -states	206
6.5	Rigid rotator	207
6.6	Energy levels and transition frequencies for a rigid rotator	209
6.7	Cylindrical coordinates	212
6.8	Energy levels of the three-dimensional harmonic oscillator	215
6.9	Levels in the spectrum of hydrogen atom	218
6.10	An electric dipole with charges $+e$ and $-e$ in an electric field gradient	218
6.11	Scheme of spin superposition in single-crystal neutron interferometry	223
6.12	Landé vectorial model for angular momentum	227

6.13	Graphical representation of the distribution of eigenvalues of the z component of the angular momenta of two independent particles	228
6.14	Angle observable and step function	241
7.1	Interferometric example of indistinguishability	246
7.2	Example of counting the number of possible configurations of bosons	253
7.3	Potential wells of a natural atom and of a quantum dot	257
8.1	Passive and active transformations	260
9.1	Representation of a measurement on the sphere of density matrices	280
9.2	Two ways of tuning the coupling function	283
9.3	Decohering histories	290
9.4	Schrödinger cat	292
9.5	Experimental realization of a Schrödinger cat with a trapped ion	293
9.6	SQUID	294
9.7	Wigner function of an entangled state	296
9.8	Schematic representation of the experiment proposed by Scully and co-workers	301
9.9	Schematic representation of Mandel's experiment	309
9.10	Interference and visibility in Mandel's experiment	310
9.11	Scully, Englert, and Walther's proposed experiment	312
9.12	Interaction-free measurement	316
9.13	Repeated interaction-free measurements	317
9.14	Probability of success in repeated interaction-free measurements	318
9.15	Interaction-free measurement with two cavities	318
9.16	Schematic representation of Mandel's experiment on empty waves	319
9.17	Depiction of Wheeler's experiment	320
9.18	Interferometry experiment for testing delayed-choice	321
9.19	Optical version of the Zeno effect	324
9.20	Example of POVMs	334
9.21	Plot of the estimate of the wave function	349
9.22	Another example of POVM	352
10.1	Stark effect	364
10.2	WKB approximation: forbidden regions outside a potential well	381
10.3	WKB and potential well	382
10.4	WKB and potential barrier	383
10.5	The different paths	390
10.6	The analogy of path integral integration	391
10.7	The sum over paths	392
10.8	Two possible paths from i to f both passing through the same central point c	394
10.9	Path integrals and scattering	397
11.1	Electron coordinates in the atomic system	406
11.2	Resulting potential in the hydrogen atom	407
11.3	Grotrian scheme	411
11.4	Plot of the radial eigenfunctions of the hydrogenoid atom	412

11.5	Plot of the radial probability densities	413
11.6	s -, p -, and d -states versus energy levels	414
11.7	Landé vectorial model for the Paschen-Bach effect	417
11.8	s and p levels in presence of the Paschen-Bach effect	418
11.9	Paschen-Bach spectroscopical lines	419
11.10	Landé vectorial model for the Zeeman effect	420
11.11	Energy levels for the Zeeman effect	422
11.12	Spectroscopical lines for the Zeeman effect	422
12.1	Spheroidal coordinates for the H_2^+ ion	440
12.2	Molecular potential energy	446
12.3	Vibrational and rotational levels of two electronic states I and II in a diatomic molecule	447
12.4	Schematic diagram of the LCAO function $f(E)$	450
12.5	LCAO energy solutions for the H_2^+ molecular ion	451
12.6	Symmetric and antisymmetric states of the ground level of the H_2^+ molecule	452
13.1	The three directions of the electromagnetic field	460
13.2	Displacement operator for coherent states	469
13.3	Phase-number uncertainty properties of coherent states	471
13.4	Phase convention for squeezed states	472
13.5	Generation of a squeezed state	473
13.6	Phase-space of amplitude- and phase-squeezed states	473
13.7	Representation of the Q-function of coherent, number, and squeezed states	475
13.8	Representation of the W-function of coherent, number, and squeezed states	480
13.9	Homodyne detection	483
13.10	Jaynes–Cummings energy levels	490
13.11	Rabi oscillations	491
13.12	Collapse and revival	492
13.13	Spontaneous and stimulated emission for a two-level atom	495
13.14	Spontaneous and stimulated emission for a three-level atom	495
13.15	Schematic diagram of a laser	495
13.16	Parametric down conversion	496
13.17	Magnetic and electric AB effects	497
13.18	Parallel transport	501
14.1	Bloch-sphere representation of states	527
14.2	Techniques for integrating a function	531
14.3	BS model for dissipation	534
14.4	Interference fringes and their sensitivity to losses in the Yurke–Stoler model	536
14.5	Pictorial representation of a coherent state and separation between the two components	537
14.6	Haroche’s experiment	538
14.7	Interference fringes in Haroche’s experiment	540
15.1	Tomographic method for reconstructing the W-function	559
15.2	Tomographic measurements of the state	560
16.1	Overview of the EPR – Bohm experiment	574

16.2	Preparation of a singlet state	576
16.3	Particle trajectories for two Gaussian slit systems after Bohm's model, and the corresponding quantum potential	580
16.4	Trajectories for a potential barrier ($E = V/2$) after Bohm's model, and the corresponding quantum potential	581
16.5	The three-dimensional Hilbert space proposed by Bell	585
16.6	Scheme of the experiment proposed for proving the second Bell theorem	589
16.7	Experiment proposed by CHSH	590
16.8	Optimal orientation for \mathbf{a} , \mathbf{a}' , \mathbf{b} , and \mathbf{b}' for testing the CHSH inequality	592
16.9	Typical dependence of $f(\theta)$ upon $n\theta$ for cases I-III	595
16.10	Partial Grotrian diagram of atomic calcium for Freedman and Clauser's experiment	599
16.11	Schematic diagram of apparatus and associated electronics of the experiment by Freedman and Clauser	599
16.12	Freedman–Clauser experiment and Aspect and co-workers' experiment	600
16.13	Alley-Shih and Ou-Mandel's experiment	602
16.14	Measured coincidence counting rate as a function of the polarizer angle θ_1 , with θ_2 fixed at 45°	604
16.15	Experimental set-up in order to solve detection loopholes	605
16.16	“Entanglement” with vacuum	606
16.17	Yurke and Stoler's experiment	608
16.18	Entanglement swapping	611
16.19	Variation of entanglement swapping	612
16.20	Orientations for the proof of Stapp's theorem	614
16.21	The GHSZ proposed experiment	616
16.22	Conditional entanglement	619
16.23	Necessary criterion for separability	623
17.1	Information difference in bits versus angle θ for the information-theoretic Bell inequality	635
17.2	Informational distance by quadrilateral inequality	635
17.3	Schematic representation of quantum non-separability	636
17.4	Diagram for entangled and disentangled states	637
17.5	Representation of all density matrices	638
17.6	Decompression of information	639
17.7	Teleportation	644
17.8	Realization of teleportation with photons	646
17.9	The CNOT gate	652
17.10	Implementation of a CNOT gate by means of a polarization interferometer	653
17.11	The quantum computation device as an equivalent of a Mach–Zender interferometer	653
17.12	Generation of Bell states by means of a Hadamard gate followed by a CNOT gate	655
17.13	Preparation of a GHSZ state	655
17.14	Toffoli gate	657

17.15	Representation of computational complexity	660
17.16	Boolean transformation of an initial bit	660
17.17	Device for solving Deutsch's problem	661
17.18	Device for solving Deutsch's problem for $n + 1$ input states	662
17.19	Device for solving Deutsch's problem for $n + m$ input states	662
17.20	Representation of Shor's theorem	664
17.21	Implementation of Grover's algorithm	666
17.22	Computational steps in Grover's algorithm	666
17.23	Classical error correction	667
17.24	Quantum circuit for error correction	669
17.25	Environmental wave functions and their overlapping as a function of time in quantum computation	670

Tables

2.1	Different cases and ways of expressing the basic quantum formalism	<i>page 71</i>
6.1	Eigenstates and eigenvalues of \hat{l}_z	199
6.2	Eigenvalues, for the three-dimensional harmonic oscillator	215
6.3	Values of j and m and the corresponding number of possible states	233
6.4	Clebsch–Gordan coefficients for $j_1 = j_2 = 1/2$	238
6.5	Clebsch–Gordan coefficients for $j_1 = 1$ and $j_2 = 1/2$	238
6.6	Clebsch–Gordan coefficients for $j_1 = j_2 = 1$	238
7.1	Fermions and bosons	250
7.2	Fermionic distributions	252
7.3	Bosonic distributions	254
11.1	Ground-state energy of helioid atoms	429
13.1	Electromagnetic spectrum	456
17.1	Sequence transmission	648
17.2	Toffoli truth table	658
17.3	Fredkin truth table	658
17.4	Classical error correction	668

Definitions, principles, etc.

Corollaries

Cor. 2.1	Simultaneous measurability	<i>page</i> 67
Cor. 16.1	Bell Dispersion-free	583

Definitions

Def. 5.1	Entanglement	183
Def. 9.1	Operation	328
Def. 16.1	Completeness	568

Lemmas

Lemma 9.1	Elby–Bub	299
Lemma 16.1	Bell I	583
Lemma 16.2	Bell II	584

Principles

Pr. 1.1	Superposition	18
Pr. 1.2	Complementarity	19
Pr. 2.1	Quantization	44
Pr. 2.2	Statistical algorithm	57
Pr. 2.3	Correspondence	72
Pr. 7.1	Symmetrization	248
Pr. 7.2	Pauli exclusion principle	251
Pr. 16.1	Separability	568
Pr. 16.2	Criterion of physical reality	569

Theorems

Th. 2.1	Hermitian operators	46
Th. 2.2	Finite-dimensional spectrum	47
Th. 2.3	Observables' equality	60
Th. 2.4	Commuting observables	66
Th. 3.1	Stone	122
Th. 5.1	Schmidt decomposition	185
Th. 6.1	Addition of angular momenta	229
Th. 7.1	Spin and statistics	249
Th. 8.1	Wigner	262

Th. 8.2	Noether	264
Th. 9.1	Kraus	329
Th. 15.1	No-Cloning	548
Th. 15.2	D'Ariano–Yuen	549
Th. 15.3	Informational completeness of unsharp observables	564
Th. 16.1	Bell I	584
Th. 16.2	Bell II	589
Th. 16.3	Stapp	613
Th. 16.4	Eberhard	620
Th. 16.5	Tsirelson	624
Th. 17.1	Lindblad	631
Th. 17.2	Holevo	642

Boxes

1.1	Interferometry	page 15
2.1	Hermitian and bounded operators	47
2.2	Example of a Hermitian operator	48
2.3	Unitary operators	51
2.4	Example of mean value	60
2.5	Commutation and product of Hermitian operators	64
2.6	Wave packet	80
3.1	Cyclic property of the trace	117
3.2	Einstein's box	132
4.1	Relativity and tunneling time	152
4.2	Scanning tunneling microscopy	152
4.3	Example of a harmonic oscillator's dynamics	163
7.1	Rasetti's discovery	250
8.1	Example of passive and active transformations	260
9.1	Decohering histories	288
9.2	Recoil-free <i>which path</i> detectors?	313
9.3	Complementarity	322
9.4	Example of postselection	326
9.5	Example of operation	327
10.1	Remarks on the Fourier transform	388
11.1	Confluent hypergeometric functions	408
13.1	LASER	494
14.1	Coherent states and macroscopic distinguishability	537
17.1	Example of factorization	663