1. Introduction.

Fourier theory is a branch of mathematics first invented to solve certain problems in partial differential equations. The most well-known of these equations are:

Laplace's equation, $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} = 0$, for $u(x, y)$ a function of two variables,

the wave equation, $\dfrac{\partial^2 u}{\partial t^2} - c^2 \dfrac{\partial^2 u}{\partial x^2} = 0$, for $u(x, t)$ a function of two variables,

the heat equation, $\dfrac{\partial u}{\partial t} - \kappa \dfrac{\partial^2 u}{\partial x^2} = 0$, for $u(x, t)$ a function of two variables.

In the heat equation, x represents the position along the bar measured from some origin, t represents time, $u(x, t)$ the temperature at position x, time t. Fourier was initially concerned with the heat equation. Incidentally, the same equation describes the concentration of a dye diffusing in a liquid such as water. For this reason the equation is sometimes called the diffusion equation.

In the wave equation, x, represents the position along an elastic string under tension, measured from some origin, t represents time, $u(x, t)$ the displacement of the string from equilibrium at position x, time t.

In Laplace's equation, $u(x, y)$ represents the steady temperature of a flat conducting plate at the position (x, y) in the plane.

Since both the heat equation and the wave equation involve a single space variable x, we sometime refer to them as the *one dimensional heat equation* and the *one dimensional wave equation* respectively.

Laplace's equation involves two spatial variables and is therefore sometimes called the *two-dimensional laplace equation*. Laplace's equation is connected to the theory of analytic functions of a complex variable. If $f(z) = u(x, y) + iv(x, y)$, the real and imaginary parts $u(x, y)$, $v(x, y)$ satisfy the Cauchy-Riemann equations,

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}, \qquad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x},$$

Then

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 v}{\partial x \partial y} = -\frac{\partial^2 u}{\partial y^2}$$

or

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0.$$

Similarly,

$$\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = 0.$$

Heat conduction and wave propagation usually occur in 3 space dimensions and are described by the following versions of Laplace's equation, the heat equation and the wave equation;

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0,$$

$$\frac{\partial u}{\partial t} - \kappa\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) = 0,$$

$$\frac{\partial^2 u}{\partial t^2} - c^2\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) = 0.$$