Preface

This book is about matrix and linear algebra, and their applications. For many students the tools of matrix and linear algebra will be as fundamental in their professional work as the tools of calculus; thus it is important to ensure that students appreciate the utility and beauty of these subjects, as well as understand the mechanics. One way to do so is to show how concepts of matrix and linear algebra make concrete problems workable. To this end, applied mathematics and mathematical modeling ought to have an important role in an introductory treatment of linear algebra.

One of the features of this book is that we weave significant motivating examples into the fabric of the text. Needless to say, I hope that instructors will not omit this material; that would be a missed opportunity for linear algebra! The text has a strong orientation towards numerical computation and applied mathematics, which means that matrix analysis plays a central role. All three of the basic components of linear algebra – theory, computation and applications – receive their due. The proper balance of these components will give a diverse audience of physical science, social science, statistics, engineering and math students the tools they need as well as the motivation to acquire these tools. Another feature of this text is an emphasis on linear algebra as an experimental science; this emphasis is to be found in certain examples, computer exercises and projects. Contemporary mathematical software makes an ideal "lab" for mathematical experimentation. At the same time, this text is independent of specific hardware and software platforms. Applications and ideas should play center stage, not software.

This book is designed for an introductory course in matrix and linear algebra. It is assumed that the student has had some exposure to calculus. Here are some of its main goals:

- To provide a balanced blend of applications, theory and computation which emphasizes their interdependence.
- To assist those who wish to incorporate mathematical experimentation through computer technology into the class. Each chapter has an optional section on computational notes and projects and computer exercises sprinkled throughout. The student should use the locally available tools to carry out the experiments suggested in the project and use the word processing capabilities of the computer system to create small reports on his/her results. In this way they gain experience in the use of the computer as a mathematical tool. One can also envision reports on a grander scale as mathematical "term papers." I have made such assignments in some of my own classes with delightful results. A few major report topics are included in the text.

- To help students to think precisely and express their thoughts clearly. Requiring written reports is one vehicle for teaching good expression of mathematical ideas. The projects given in this text provide material for such reports.
- To encourage cooperative learning. Mathematics educators are becoming increasingly appreciative of this powerful mode of learning. Team projects and reports are excellent vehicles for cooperative learning.
- To promote individual learning by providing a complete and readable text. I hope that students will find the text worthy of being a permanent part of their reference library, particularly for the basic linear algebra needed for the applied mathematical sciences.

An outline of the book is as follows: Chapter 1 contains a thorough development of Gaussian elimination and an introduction to matrix notation. It would be nice to assume that the student is familiar with complex numbers, but experience has shown that this material is frequently long forgotten by many. Complex numbers and the basic language of sets are reviewed early on in Chapter 1. (The advanced part of the complex number discussion could be deferred until it is needed in Chapter 4.) In Chapter 2, basic properties of matrix and determinant algebra are developed. Special types of matrices, such as elementary and symmetric, are also introduced. About determinants: some instructors prefer not to spend too much time on them, so I have divided the treatment into two sections, one of which is marked as optional and not used in the rest of the text. Chapter 3 begins by introducing the student to the "standard" Euclidean vector spaces, both real and complex. These are the well springs for the more sophisticated ideas of linear algebra. At this point the student is introduced to the general ideas of abstract vector space, subspace and basis, but primarily in the context of the standard spaces. Chapter 4 introduces goemetrical aspects of standard vectors spaces such as norm, dot product and angle. Chapter 5 provides an introduction to eigenvalues and eigenvectors. Subsequently, general norm and inner product concepts are examined in Chapter 5. Two appendices are devoted to a table of commonly used symbols and solutions to selected exercises.

Each chapter contains a few more "optional" topics, which are independent of the non-optional sections. I say this realizing full well that one instructor's optional is another's mandatory. Optional sections cover tensor products, linear operators, operator norms, the Schur triangularization theorem and the singular value decomposition. In addition, each chapter has an optional section of computational notes and projects. I have employed the convention of marking sections and subsections that I consider optional with an asterisk. Finally, at the end of each chapter is a selection of review exercises.

There is more than enough material in this book for a one semester course. Tastes vary, so there is ample material in the text to accommodate different interests. One could increase emphasis on any one of the theoretical, applied or computational aspects of linear algebra by the appropriate selection of syllabus topics. The text is well suited to a course with a three hour lecture and lab component, but the computer related material is not mandatory. Every instructor has her/his own idea about how much time to spend on proofs, how much on examples, which sections to skip, etc.; so the amount of material covered will vary considerably. Instructors may mix and match any of the optional sections according to their own interests, since these sections are largely independent of each other. My own opinion is that the ending sections in each chapter on computational notes and projects are partly optional. While it would be very time consuming to cover them all, every instructor ought to use some part of this material. The unstarred sections form the core of the book; most of this material should be covered. There are 27 unstarred sections and 12 optional sections. I hope the optional sections come in enough flavors to please any pure, applied or computational palate.

Of course, no one shoe size fits all, so I will suggest two examples of how one might use this text for a three hour one semester course. Such a course will typically meet three times a week for fifteen weeks, for a total of 45 classes. The material of most of the the unstarred sections can be covered at a rate of about one and one half class periods per section. Thus, the core material could be covered in about 40 class periods. This leaves time for extra sections and and in-class exams. In a two semester course or a semester course of more than three hours, one could expect to cover most, if not all, of the text.

If the instructor prefers a course that emphasizes the standard Euclidean spaces, and moves at a more leisurely pace, then the core material of the first five chapters of the text are sufficient. This approach reduces the number of unstarred sections to be covered from 27 to 23.

In addition to the usual complement of pencil and paper exercises (with selected solutions in Appendix B), this text includes a number of computer related activities and topics. I employ a taxonomy for these activities which is as follows. At the lowest level are computer exercises. Just as with pencil and paper exercises, this work is intended to develop basic skills. The difference is that some computing equipment (ranging from a programmable scientific calculator to a workstation) is required to complete such exercises. At the next level are computer projects. These assignments involve ideas that extend the standard text material, possibly some experimentation and some written exposition in the form of brief project papers. These are analogous to lab projects in the physical sciences. Finally, at the top level are reports. These require a more detailed exposition of ideas, considerable experimentation – possibly open ended in scope, and a carefully written report document. Reports are comparable to “scientific term papers”. They approximate the kind of activity that many students will be involved in through their professional life. I have included some of my favorite examples of all three activities in this textbook. Exercises that require computing tools contain a statement to that effect. Perhaps projects and reports I have included will be paradigms for instructors who wish to build their own project/report materials. In my own classes I expect projects to be prepared with text processing software to which my students have access in a mathematics computer lab.

Projects and reports are well suited for team efforts. Instructors should provide background materials to help the students through local system dependent issues. For example, students in my own course are assigned a computer account in the mathematics lab and required to attend an orientation that contains specific information about the available linear algebra software. When I assign a project, I usually make available a Maple or Mathematica notebook that amounts to a brief background lecture on the subject of the project and contains some of the key commands students will need to carry out the project. This helps students focus more on the mathematics of the project rather than computer issues.

Most of the computational computer tools that would be helpful in this course fall into three categories and are available for many operating systems:

- Graphing calculators with built-in matrix algebra capabilities such as the HP 28 and 48, or the TI 85 and 92. These use floating point arithmetic for system solving and matrix arithmetic. Some do eigenvalues.
- Computer algebra systems (CAS) such as Maple, Mathematica and Macsyma. These software products are fairly rich in linear algebra capabilities. They prefer symbolic calculations and exact arithmetic, but will do floating point calculations, though some coercion may be required.
- Matrix algebra systems (MAS) such as MATLAB or Octave. These software products are specifically designed to do matrix calculations in floating point arithmetic, though limited symbolic capabilities are available in the basic program. They have the most complete set of matrix commands of all categories.

In a few cases I have included in this text some software specific information for some projects, for the purpose of illustration. This is not to be construed as an endorsement or requirement of any particular software or computer. Projects may be carried out with different software tools and computer platforms. Each system has its own strengths. In various semesters I have obtained excellent results with all these platforms. Students are open to all sorts of technology in mathematics. This openness, together with the availability of inexpensive high technology tools, is changing how and what we teach in linear algebra.

I would like to thank my colleagues whose encouragement has helped me complete this project, particularly Jamie Radcliffe, Jim Lewis, Dale Mesner and John Bakula. Special thanks also go to Jackie Kohles for her excellent work on solutions to the exercises and to the students in my linear algebra courses for relentlessly tracking down errors. I would also like to thank my wife, Muriel, for an outstanding job of proofreading and editing the text.

I'm in the process of developing a linear algebra home page of material such as project notebooks, supplementary exercises, etc, that will be useful for instructors and students of this course. This site can be reached through my home page at

`http://www.math.unl.edu/~tshores/`

I welcome suggestions, corrections or comments on the site or book; both are ongoing projects. These may be sent to me at `tshores@math.unl.edu`.

Preface

This book is about matrix and linear algebra, and their applications. For many students the tools of matrix and linear algebra will be as fundamental in their professional work as the tools of calculus; thus it is important to ensure that students appreciate the utility and beauty of these subjects, as well as understand the mechanics. One way to do so is to show how concepts of matrix and linear algebra make concrete problems workable. To this end, applied mathematics and mathematical modeling ought to have an important role in an introductory treatment of linear algebra.

One of the features of this book is that we weave significant motivating examples into the fabric of the text. Needless to say, I hope that instructors will not omit this material; that would be a missed opportunity for linear algebra! The text has a strong orientation towards numerical computation and applied mathematics, which means that matrix analysis plays a central role. All three of the basic components of linear algebra – theory, computation and applications – receive their due. The proper balance of these components will give a diverse audience of physical science, social science, statistics, engineering and math students the tools they need as well as the motivation to acquire these tools. Another feature of this text is an emphasis on linear algebra as an experimental science; this emphasis is to be found in certain examples, computer exercises and projects. Contemporary mathematical software makes an ideal "lab" for mathematical experimentation. At the same time, this text is independent of specific hardware and software platforms. Applications and ideas should play center stage, not software.

This book is designed for an introductory course in matrix and linear algebra. It is assumed that the student has had some exposure to calculus. Here are some of its main goals:

- To provide a balanced blend of applications, theory and computation which emphasizes their interdependence.
- To assist those who wish to incorporate mathematical experimentation through computer technology into the class. Each chapter has an optional section on computational notes and projects and computer exercises sprinkled throughout. The student should use the locally available tools to carry out the experiments suggested in the project and use the word processing capabilities of the computer system to create small reports on his/her results. In this way they gain experience in the use of the computer as a mathematical tool. One can also envision reports on a grander scale as mathematical "term papers." I have made such assignments in some of my own classes with delightful results. A few major report topics are included in the text.

- To help students to think precisely and express their thoughts clearly. Requiring written reports is one vehicle for teaching good expression of mathematical ideas. The projects given in this text provide material for such reports.
- To encourage cooperative learning. Mathematics educators are becoming increasingly appreciative of this powerful mode of learning. Team projects and reports are excellent vehicles for cooperative learning.
- To promote individual learning by providing a complete and readable text. I hope that students will find the text worthy of being a permanent part of their reference library, particularly for the basic linear algebra needed for the applied mathematical sciences.

An outline of the book is as follows: Chapter 1 contains a thorough development of Gaussian elimination and an introduction to matrix notation. It would be nice to assume that the student is familiar with complex numbers, but experience has shown that this material is frequently long forgotten by many. Complex numbers and the basic language of sets are reviewed early on in Chapter 1. (The advanced part of the complex number discussion could be deferred until it is needed in Chapter 4.) In Chapter 2, basic properties of matrix and determinant algebra are developed. Special types of matrices, such as elementary and symmetric, are also introduced. About determinants: some instructors prefer not to spend too much time on them, so I have divided the treatment into two sections, one of which is marked as optional and not used in the rest of the text. Chapter 3 begins by introducing the student to the "standard" Euclidean vector spaces, both real and complex. These are the well springs for the more sophisticated ideas of linear algebra. At this point the student is introduced to the general ideas of abstract vector space, subspace and basis, but primarily in the context of the standard spaces. Chapter 4 introduces goemetrical aspects of standard vectors spaces such as norm, dot product and angle. Chapter 5 provides an introduction to eigenvalues and eigenvectors. Subsequently, general norm and inner product concepts are examined in Chapter 5. Two appendices are devoted to a table of commonly used symbols and solutions to selected exercises.

Each chapter contains a few more "optional" topics, which are independent of the non-optional sections. I say this realizing full well that one instructor's optional is another's mandatory. Optional sections cover tensor products, linear operators, operator norms, the Schur triangularization theorem and the singular value decomposition. In addition, each chapter has an optional section of computational notes and projects. I have employed the convention of marking sections and subsections that I consider optional with an asterisk. Finally, at the end of each chapter is a selection of review exercises.

There is more than enough material in this book for a one semester course. Tastes vary, so there is ample material in the text to accommodate different interests. One could increase emphasis on any one of the theoretical, applied or computational aspects of linear algebra by the appropriate selection of syllabus topics. The text is well suited to a course with a three hour lecture and lab component, but the computer related material is not mandatory. Every instructor has her/his own idea about how much time to spend on proofs, how much on examples, which sections to skip, etc.; so the amount of material covered will vary considerably. Instructors may mix and match any of the optional sections according to their own interests, since these sections are largely independent of each other. My own opinion is that the ending sections in each chapter on computational notes and projects are partly optional. While it would be very time consuming to cover them all, every instructor ought to use some part of this material. The unstarred sections form the core of the book; most of this material should be covered. There are 27 unstarred sections and 12 optional sections. I hope the optional sections come in enough flavors to please any pure, applied or computational palate.

Of course, no one shoe size fits all, so I will suggest two examples of how one might use this text for a three hour one semester course. Such a course will typically meet three times a week for fifteen weeks, for a total of 45 classes. The material of most of the the unstarred sections can be covered at a rate of about one and one half class periods per section. Thus, the core material could be covered in about 40 class periods. This leaves time for extra sections and and in-class exams. In a two semester course or a semester course of more than three hours, one could expect to cover most, if not all, of the text.

If the instructor prefers a course that emphasizes the standard Euclidean spaces, and moves at a more leisurely pace, then the core material of the first five chapters of the text are sufficient. This approach reduces the number of unstarred sections to be covered from 27 to 23.

In addition to the usual complement of pencil and paper exercises (with selected solutions in Appendix B), this text includes a number of computer related activities and topics. I employ a taxonomy for these activities which is as follows. At the lowest level are computer exercises. Just as with pencil and paper exercises, this work is intended to develop basic skills. The difference is that some computing equipment (ranging from a programmable scientific calculator to a workstation) is required to complete such exercises. At the next level are computer projects. These assignments involve ideas that extend the standard text material, possibly some experimentation and some written exposition in the form of brief project papers. These are analogous to lab projects in the physical sciences. Finally, at the top level are reports. These require a more detailed exposition of ideas, considerable experimentation – possibly open ended in scope, and a carefully written report document. Reports are comparable to “scientific term papers”. They approximate the kind of activity that many students will be involved in through their professional life. I have included some of my favorite examples of all three activities in this textbook. Exercises that require computing tools contain a statement to that effect. Perhaps projects and reports I have included will be paradigms for instructors who wish to build their own project/report materials. In my own classes I expect projects to be prepared with text processing software to which my students have access in a mathematics computer lab.

Projects and reports are well suited for team efforts. Instructors should provide background materials to help the students through local system dependent issues. For example, students in my own course are assigned a computer account in the mathematics lab and required to attend an orientation that contains specific information about the available linear algebra software. When I assign a project, I usually make available a Maple or Mathematica notebook that amounts to a brief background lecture on the subject of the project and contains some of the key commands students will need to carry out the project. This helps students focus more on the mathematics of the project rather than computer issues.

Most of the computational computer tools that would be helpful in this course fall into three categories and are available for many operating systems:

- Graphing calculators with built-in matrix algebra capabilities such as the HP 28 and 48, or the TI 85 and 92. These use floating point arithmetic for system solving and matrix arithmetic. Some do eigenvalues.
- Computer algebra systems (CAS) such as Maple, Mathematica and Macsyma. These software products are fairly rich in linear algebra capabilities. They prefer symbolic calculations and exact arithmetic, but will do floating point calculations, though some coercion may be required.
- Matrix algebra systems (MAS) such as MATLAB or Octave. These software products are specifically designed to do matrix calculations in floating point arithmetic, though limited symbolic capabilities are available in the basic program. They have the most complete set of matrix commands of all categories.

In a few cases I have included in this text some software specific information for some projects, for the purpose of illustration. This is not to be construed as an endorsement or requirement of any particular software or computer. Projects may be carried out with different software tools and computer platforms. Each system has its own strengths. In various semesters I have obtained excellent results with all these platforms. Students are open to all sorts of technology in mathematics. This openness, together with the availability of inexpensive high technology tools, is changing how and what we teach in linear algebra.

I would like to thank my colleagues whose encouragement has helped me complete this project, particularly Jamie Radcliffe, Jim Lewis, Dale Mesner and John Bakula. Special thanks also go to Jackie Kohles for her excellent work on solutions to the exercises and to the students in my linear algebra courses for relentlessly tracking down errors. I would also like to thank my wife, Muriel, for an outstanding job of proofreading and editing the text.

I'm in the process of developing a linear algebra home page of material such as project notebooks, supplementary exercises, etc, that will be useful for instructors and students of this course. This site can be reached through my home page at

`http://www.math.unl.edu/~tshores/`

I welcome suggestions, corrections or comments on the site or book; both are ongoing projects. These may be sent to me at `tshores@math.unl.edu`.