
Table of Contents

1	Introduction	1
1.1	Motivation	1
1.2	Contributions and Outline	4
2	Foundation	7
2.1	Operations Research	7
2.1.1	Graph Theory	7
2.1.2	Linear Programming	12
2.1.3	Semidefinite Programming	14
2.2	Constraint Programming	15
2.2.1	Basic Notions	15
2.2.2	Propagation	16
2.2.3	Search	17
<hr/>		
	Part I: Propagation	25
<hr/>		
3	A Systematic Overview of the Alldifferent Constraint	27
3.1	Introduction	27
3.2	Combinatorial Background	29
3.2.1	Alldifferent and Bipartite Matching	29
3.2.2	Hall's Marriage Theorem	30
3.3	Local Consistency Notions	31
3.4	Propagation for Local Consistency Notions	33
3.4.1	Local Consistency of a Decomposed CSP	33
3.4.2	Bounds Consistency	36
3.4.3	Range Consistency	38
3.4.4	Hyper-arc Consistency	39
3.4.5	Complexity Survey and Discussion	42
3.5	Variants of the Alldifferent Constraint	43
3.5.1	The Symmetric Alldifferent Constraint	43

3.5.2	The Weighted Alldifferent Constraint	46
3.6	The Alldifferent Polytope	50
3.7	Conclusion	52
4	Soft Global Constraints	53
4.1	Introduction	53
4.2	Related Literature	55
4.3	Outline of Method	56
4.3.1	Constraint Softening and Violation Measures	56
4.3.2	Propagation of Soft Constraints	58
4.4	Soft Alldifferent Constraint	60
4.4.1	Definitions	60
4.4.2	Graph Representation	61
4.4.3	Variable-Based Violation Measure	62
4.4.4	Decomposition-Based Violation Measure	63
4.5	Soft Global Cardinality Constraint	65
4.5.1	Definitions	65
4.5.2	Graph Representation	68
4.5.3	Variable-Based Violation Measure	69
4.5.4	Value-Based Violation Measure	70
4.6	Soft Regular Constraint	72
4.6.1	Definitions	72
4.6.2	Graph Representation	74
4.6.3	Variable-Based Violation Measure	75
4.6.4	Edit-Based Violation Measure	76
4.7	Soft Same Constraint	78
4.7.1	Definitions	78
4.7.2	Graph Representation	79
4.7.3	Variable-Based Violation Measure	80
4.8	Aggregating Soft Constraints	81
4.9	Conclusion	82
Part II: Search		85
5	Postponing Branching Decisions	87
5.1	Introduction	87
5.2	Outline of Method	88
5.3	Theoretical Analysis	90
5.4	Computational Results	93
5.4.1	Travelling Salesman Problem	95
5.4.2	Partial Latin Square Completion Problem	96
5.5	Discussion and Conclusion	98

6 Reduced Costs as Branching Heuristic	99
6.1 Introduction	99
6.2 Solution Framework	101
6.2.1 Building a Linear Programming Relaxation	101
6.2.2 Domain Partitioning using Reduced Costs	102
6.2.3 Discrepancy Constraint	104
6.3 Discrepancy-Based Bound Improvement	105
6.4 The Travelling Salesman Problem	107
6.4.1 Constraint Programming Model	107
6.4.2 Integer Linear Programming Model	109
6.4.3 Linear Programming Relaxation	110
6.5 Computational Results	111
6.5.1 Implementation	111
6.5.2 Quality of Heuristic	111
6.5.3 Symmetric TSP Instances	114
6.5.4 Asymmetric TSP with Time Windows Instances	116
6.6 Discussion and Conclusion	117
7 Semidefinite Relaxation as Branching Heuristic	119
7.1 Introduction	119
7.2 Motivation	120
7.3 Solution Framework	120
7.3.1 Building a Semidefinite Relaxation	120
7.3.2 Applying the Semidefinite Relaxation	122
7.4 The Stable Set Problem	123
7.4.1 Integer Programming Models	123
7.4.2 Semidefinite Programming Relaxation	124
7.5 Computational Results	126
7.5.1 Implementation	126
7.5.2 Characterization of Problem Instances	126
7.5.3 Random Weighted and Unweighted Graphs	128
7.5.4 Graphs Arising from Coding Theory	128
7.5.5 Graphs from the DIMACS Benchmarks Set	130
7.6 Discussion and Conclusion	133
Perspectives	135
References	137
Index	149
Samenvatting	153