Preface

These are the lecture notes from a course I gave at Berkeley in the spring of 1989. My original motivation was to understand the formulae for the "weights" (i.e., Fourier coefficients) of the Markov trace which were computed by Ocneanu **[3]**. Luckily for me, halfway through the course Vaughan Jones showed me a slick preprint by Springer **[9]** which gave a very simple explanation of Ocneanu's results, and I have incorporated this approach in §14. The prerequisites for this are §1 and most of Part II. But it is important to point out that one needs to know almost nothing in order to prove that the trace exists and to obtain a constructive algorithm for calculating it. The reader who is only interested in this aspect can just read (13.1), (13.2), (13.3), and (14.1), and should definitely see **[5]**.

The course was organized into three parts, as is clear from the table of contents. In Part I, I prove Burnside's Theorem that groups of order $p^a q^b$ are solvable (3.6), Frobenius' Theorem on the existence of Frobenius kernels (4.5), and Brauer's characterization of characters (5.8). As an application of the latter, I prove Brauer's theorem on blocks of defect zero (5.11). The reader who is only interested in the later material can skip all of §5 and most, if not all, of §3. The most important results which are needed in Part II are the basic facts about induced characters, primarily Mackey's theorem (4.4).

The material in Part II is far from complete, the most glaring omission being the Littlewood-Richardson rule. I first give an algorithm for computing the character table of S^n (§7) and I construct the Specht modules (§8) following James **[4]**. Following Macdonald **[8]**, I next derive the "determinant form" (11.4) for the irreducible characters of S^n using the theory of symmetric functions, and I then obtain the hook-length formula (12.1) and the Murnaghan-Nakayama formula (12.6) as consequences.

In Part III, I prove that the field of rational functions is a splitting field for the Hecke algebra by first extending scalars to the field of formal Laurent series and then descending. The reader who is content to just use Laurent series can skip this and save a little time. I then develop Springer's observation that the Fourier transform of the Markov trace is really a homomorphism from the ring of symmetric functions to the field of rational functions in two variables.