

# PREFACE

The theory of loops (groups without associativity), though researched by several mathematicians has not found a sound expression, for books, be it research level or otherwise, solely dealing with the properties of loops are absent. This is in marked contrast with group theory where books are abundantly available for all levels: as graduate texts and as advanced research books.

The only three books available where the theory of loops are dealt with are: R. H. Bruck, *A Survey of Binary Systems*, Springer Verlag, 1958 recent edition (1971); H. O. Pflugfelder, *Quasigroups and Loops: Introduction*, Heldermann Verlag, 1990; Orin Chein, H. O. Pflugfelder and J.D.H. Smith (editors), *Quasigroups and Loops: Theory and Applications*, Heldermann Verlag, 1990. But none of them are completely devoted for the study of loops.

The author of this book has been working in loops for the past 12 years, and has guided a Ph.D. and 3 post-graduate research projects in this field of loops feels that the main reason for the absence of books on loops is the fact that it is more abstract than groups. Further one is not in a position to give a class of loops which are as concrete as the groups of the form  $S_n$ ,  $D_{2n}$  etc. which makes the study of these non-associative structures much more complicated. To overcome this problem in 1994 the author with her Ph.D. student S. V. Singh has introduced a new class of loops using modulo integers. They serve as a concrete examples of loops of even order and it finds an application to colouring of the edges of the graph  $K_{2n}$ .

Several researchers like Bruck R. H., Chibuka V. O., Doro S., Giordano G., Glauberman G., Kunen K., Liebeck M.W., Mark P., Michael Kinyon, Orin Chein, Paige L.J., Pflugfelder H.O., Phillips J.D., Robinson D. A., Solarin A. R. T., Tim Hsu, Wright C.R.B. and by others who have worked on Moufang loops and other loops like Bol loops, A-loops, Steiner loops and Bruck loops. But some of these loops become Moufang loops. Orin Chein, Michael Kinyon and others have studied loops and the Lagrange property.

The purpose of this book entirely lies in the study, introduction and examination of the Smarandache loops based on a paper about *Special Algebraic Structures* by Florentin Smarandache. As a result, this book doesn't give a full-fledged analysis on loops and their properties. However, for the sake of readers who are involved in the study of loop theory we have provided a wide-ranging list of papers in the reference. We expect the reader to have a good background in algebra and more specifically a strong foundation in loops and number theory.

This book introduces over 75 Smarandache concepts on loops, and most of these concepts are illustrated by examples. In fact several of the Smarandache loops have classes of loops which satisfy the Smarandache notions.

This book is structured into five chapters. Chapter one which is introductory in nature covers some notions about groups, graphs and lattices. Chapter two gives some basic properties of loops. The importance of this chapter lies in the introduction of a new class of loops of even order. We prove that the number of different representations of right alternative loop of even order  $(2n)$ , in which square of each element is identity is equal to the number of distinct proper  $(2n - 1)$  edge colourings of the complete graph  $K_{2n}$ .

In chapter three we introduce Smarandache loops and their Smarandache notions. Except for the Smarandache notions several of the properties like Lagrange's criteria, Sylow's criteria may not have been possible. Chapter four introduces Smarandache mixed direct product of loops which enables us to define a Smarandache loops of level II and this class of loops given by Smarandache mixed direct product gives more concrete and non-abstract structure of Smarandache loops in general and loops in particular. The final section gives 52 research problems for the researchers in order to make them involved in the study of Smarandache loops. The list of problems provided at the end of each section is a main feature of this book.

I deeply acknowledge the encouragement that Dr. Minh Perez extended to me during the course of this book. It was because of him that I got started in this endeavour of writing research books on Smarandache algebraic notions.

I dedicate this book to my parents, Worathur Balasubramanian and Krishnaveni for their love.