Preface

The development of *computer aided engineering design* has gained momentum over the last three decades. Computer graphics, geometric modeling of curves, surfaces and solids, finite element method, optimization, computational fluid flow and heat transfer—all have now taken roots into the academic curricula as individual disciplines. Several professional softwares are now available for the design of surfaces and solids. These are very user-friendly and do not require a user to possess the intricate details of the mathematical basis that goes behind.

This book is an outcome of over a decade of teaching computer aided design to graduate and senior undergraduate students. It emphasizes the mathematical background behind geometric modeling, analysis and optimization tools incorporated within the existing software.

- Much of the material on CAD related topics is widely scattered in literature. This book is conceived with a view to arrange the source material in a logical and comprehensive sequence, to be used as a semester course text for CAD.
- The *focus* is on computer aided design. Treatment essential for geometric transformations, projective geometry, differential geometry of curves and surfaces have been dealt with in detail using examples. Only a background in elementary linear algebra, matrices and vector geometry is required to understand the material presented.
- The concepts of homogeneous transformations and affine spaces (barycentric coordinate system) have been explained with examples. This is essential to understand how a solid or surface model of an object can escape coordinate system dependence. This enables a distortion-free handling of a computer model under rigid-body transformations.
- A viewpoint that free-form solids may be regarded as composed of surface patches which instead are composed of curve segments is maintained in this book, like most other texts on CAD. Thus, geometric modeling of curve segments is discussed in detail. The basis of curve design is parametric, piecewise fitting of individual segments of low degree into a composite curve such that the desired continuity (position, slope and/or curvature) is maintained between adjacent segments. This reduces undue oscillations and provides freedom to a designer to alter the curve shape. A generic model of a curve segment is the weighted linear combination of user-specified data points where the weights are functions of a *normalized*, *non-negative* parameter. Further, barycentricity of weights* makes a curve segment independent of the coordinate system and provides an insight into the curve's shape. That is, the curve lies within

* Weights are all non-negative and for any value of the parameter, they sum to unity.

the convex hull of the data points specified. The associated variation diminishing property suggests that the curve's shape is no more complex than the polyline of the control points itself. In other words, a control polyline primitively approximates the shape of the curve. For Bézier segments, barycentricity is global in that altering any data point results in overall shape change of the segment. For B-spline curves, however, weights are locally barycentric allowing shape change only within some local region. Expressions for weights, that is, Bernstein polynomials for Bézier segments and B-spline basis functions for B-spline curves are derived and discussed in detail in this book and many examples are presented to illustrate curve design.

- With the design of free-form curve segments accomplished, surface patches can be obtained in numerous ways. With two curves, one can sweep one over the other to get a *sweep surface patch*. One of the curves can be rectilinear in shape and represent an axis about which the second curve can be revolved to get a *patch of revolution*. One can join corresponding points on the two curves using straight lines to generate a *ruled surface*. Or, if cross boundary slope information is available, one can join the corresponding points using a cubic segment to get a *lofted patch*. More involved models of surface patches are the bilinear and bicubic Coon's patches wherein four boundary curves are involved. Eventually, a direct extension of Bézier and B-spline curves is their tensor product into respective free-form Bézier and B-spline surface patches. These surface patches inherit the properties from the respective curves. That is, the surface patch lies within the control polyhedron defined by the data points, and that the polyhedron loosely represents the patch shape. The aforementioned patches are derived and discussed in detail with examples in this book. Later, methods to model composite surfaces are discussed.
- The basis for solid modeling is the extension of Jordon's curve theorem which states that a closed, simply connected** (planar) curve divides a plane into two regions; its interior and its exterior. Likewise, a closed, simply connected and orientable surface divides a three-dimensional space into regions interior and exterior to the surface. With this established, a simple, closed and connected surface constituted of various surface patches *knit* or *glued* together at their respective common boundaries encloses a finite volume within itself. The union of this interior region with the surface boundary represents a free form solid. Any solid modeler should be generic and capable of modeling unambiguous solids such that any set operation (union, intersection or difference) performed on two valid solids should yield another valid solid. With this viewpoint, the concept of geometry is relaxed to study the topological attributes of valid solids. Such properties disregard *size* (lengths and angles) and study only the *connectivity* in a solid. With these properties as basis, the three solid modeling techniques, i.e., wireframe modeling, boundary representation method and constructive solid geometry are discussed in detail with examples. Advantages and drawbacks of each method are discussed and it is emphasized that professional solid modelers utilize all three representations depending on the application. For instance, wireframe modeling is usually employed for animation as quick rendering is not possible with the boundary representation scheme.
- Determination of intersection between various curves, surfaces and solids is routinely performed by the solid modelers for curve and surface trimming and blending. Intersection determination is primarily used in computing Boolean relations between two solids in constructive solid

** A closed curve with no self intersection.

geometry. Computational geometry that encompasses a set of algorithms to compute various relations like proximity, intersection, decomposition and relational search (e.g., point membership classification) between geometric entities is discussed in brief in this book. The working of these algorithms is described for polygonal entities with examples for easy understanding of the subject matter.

- Reverse engineering alludes to the process of creating CAD models from existing real life components or their prototypes. Applications are prolific; some being the generation of customized fit to human surfaces, designing prostheses, and reconstruction of archaeological collections and artifacts. For an engineering component whose original data is not available, a conceptual clay or wood model is employed. A point cloud data is acquired from an existing component or its prototype using available non-contact or tactile scanning methods. Surface patches are then locally modeled over a subset of the point cloud to interpolate or best approximate the data. Reverse engineering is an important emerging application in Computer Aided Design, and various methods for surface patch fitting, depending on the scanning procedure used, are briefed in this book.
- Having discussed in detail the geometric modeling aspects in free-form design, this book provides an introductory treatment to the finite element analysis (FEM) and optimization, the other two widely employed tools in computer aided design. Using these, one can analyze and alter a design form such that the latter becomes optimal in some sense of the user specified objective. The book discusses linear elastic finite element method using some basic elements like trusses, frames, triangular and four-node elements. Discussion on optimization is restricted to some numerical methods in determining single variable extrema and classical Karush-Kuhn-Tucker necessary conditions for multi-variable unconstrained and constrained problems. Sequential Linear and Quadratic Programming, and stochastic methods like genetic algorithms and simulated annealing are given a brief mention. The intent is to introduce a student to follow-up formal courses on finite element analysis and optimization in the curricula.

This book should be used by the educators as follows:

Students from a variety of majors, e.g., mechanical engineering, computer science and engineering, aeronautical and civil engineering and mathematics are likely to credit this course. Also, students may study CAD at primarily graduate and senior undergraduate levels. Geometric modeling of curves, surfaces and solids may be relevant to all while finite element analysis and optimization may be of interest of mechanical, aeronautical and civil engineering. Discretion of the instructor may be required to cover the combination of topics for a group of students. Considering a semester course of 40 contact hours, a broad breakup of topics is suggested as follows:

- 1st hour: Introduction to computer aided design
- 3 hours: Transformations and projections
- 15 hours: Free-form curve design
- 9 hours: Surface patch modeling
- 6 hours: Solid modeling

The remaining 6 hours may be assigned as follows: for students belonging to mechanical, aeronautical and civil engineering, reverse engineering, finite element method and optimization may be introduced and for those in computer science and engineering and mathematics, computational geometry and optimization may be emphasized.

For a group of graduate students taking this course, differential geometry of curves and surfaces

(Chapters 3 and 6) may be dealt with in detail. Also, topological attributes of solids may be discussed. For only senior undergraduate students, differential geometry may be covered in brief emphasizing mainly Frenet-Serret relations, Gaussian and Mean curvatures and their importance in determining the nature of a surface. Chapters on computational geometry, reverse engineering, FEM and optimization may be omitted.

Assignments and projects form an important part of this course. Assignments may be tailored in a manner that students get a handle on manual calculations as well as code development for curve and surface design. A course project may run over a semester or can be in two parts each covering half the semester. Some example projects are mentioned in Appendix III.

Some examples presented in Chapter 1 on kinematic analysis and spring design pertain to students in mechanical engineering. For a generic class, an instructor may prefer to cover curve interpolation and fitting discussed in sections 3.1 and 3.2.

The practitioners, i.e., those developing professional software would require much deeper understanding of the design principles, mathematical foundations and computer graphics to render a robust Graphical User Interface to the software. This book would help them acquire adequate background knowledge in design principles and mathematical foundations. Those using the software may not require a deeper understanding of the mathematical principles. However, design aspects and essential properties of curve, surface and solid modeling would be needed to create the design and interpret the results.

Chapters 9 and 10 of this book on computations with geometry and modeling using point clouds has been contributed by Dr. G. Saravana Kumar, a former Ph.D. Student, Mechanical Engineering Department, IIT Kanpur. His enthusiasm as T.A. in the CAD course has also resulted in several good projects.

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