Preface

In high school algebraic equations in one unknown of first and second degree are studied in detail. One learns that for solving these equations there exist general formulae expressing their roots in terms of the coefficients by means of arithmetic operations and of radicals. But very few students know whether similar formulae do exist for solving algebraic equations of higher order. In fact, such formulae also exist for equations of the third and fourth degree. We shall illustrate the methods for solving these equations in the introduction. Nevertheless, if one considers the generic equation in one unknown of degree higher than four one finds that it is not solvable by radicals: there exist no formulae expressing the roots of these equations in terms of their coefficients by means of arithmetic operations and of radicals. This is exactly the statement of the Abel theorem.

One of the aims of this book is to make known this theorem. Here we will not consider in detail the results obtained a bit later by the French mathematician Évariste Galois. He considered some special algebraic equation, i.e., having particular numbers as coefficients, and for these equations found the conditions under which the roots are representable in terms of the coefficients by means of algebraic equations and radicals[1].

From the general Galois results one can, in particular, also deduce the Abel theorem. But in this book we proceed in the opposite direction: this will allow the reader to learn two very important branches of modern mathematics: group theory and the theory of functions of one complex variable. The reader will be told what is a group (in mathematics), a field, and which properties they possess. He will also learn what the complex numbers are and why they are defined in such a manner and not

[1] To those who wish to learn the Galois results we recommend the books: Postnikov M.M., Boron L.F., Galois E., *Fundamentals of Galois Theory,* (Nordhoff: Groningen), (1962); Van der Waerden B.L., Artin E., Noether E., Algebra, (Ungar: New York, N.Y.) (1970).

otherwise. He will learn what a Riemann surface is and of what the 'basic theorem of the complex numbers algebra' consists.

The author will accompany the reader along this path, but he will also give him the possibility of testing his own forces. For this purpose he will propose to the reader a large number of problems. The problems are posed directly within the text, so representing an essential part of it. The problems are labelled by increasing numbers in bold figures. Whenever the problem might be too difficult for the reader, the chapter 'Hint, Solutions, and Answers' will help him.

The book contains many notions which may be new to the reader. To help him in orienting himself amongst these new notions we put at the end of the book an alphabetic index of notions, indicating the pages where their definitions are to be found.

The proof of the Abel theorem presented in this book was presented by professor Vladimir Igorevich Arnold during his lectures to the students of the la 11th course of the physics-mathematics school of the State University of Moscow in the years 1963–64. The author of this book, who at that time was one of the pupils of that class, during the years 1970–71 organized for the pupils of that school a special seminar dedicated to the proof of the Abel theorem. This book consists of the material collected during these activities. The author is very grateful to V.I. Arnold for having made a series of important remarks during the editing of the manuscript.

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