Preface

Some time ago Charles B. Morrey and I wrote *A First Course in Real Analysis*, a book that provides material sufficient for a comprehensive one-year course in analysis for those students who have completed a standard elementary course in calculus. The book has been through two editions, the second of which appeared in 1991; small changes and corrections of misprints have been made in the fifth printing of the second edition, which appeared recently.

However, for many students of mathematics and for those students who intend to study any of the physical sciences and computer science, the need is for a short one-semester course in real analysis rather than a lengthy, detailed, comprehensive treatment. To fill this need the book *Basic Elements of Real Analysis* provides, in a brief and elementary way, the most important topics in the subject.

The first chapter, which deals with the real number system, gives the reader the opportunity to develop facility in proving elementary theorems. Since most students who take this course have spent their efforts in developing manipulative skills, such an introduction presents a welcome change. The last section of this chapter, which establishes the technique of mathematical induction, is especially helpful for those who have not previously been exposed to this important topic.

Chapters 2 through 5 cover the theory of elementary calculus, including differentiation and integration. Many of the theorems that are "stated without proof" in elementary calculus are proved here.

It is important to note that both the Darboux integral and the Riemann integral are described thoroughly in Chapter 5 of this volume. Here we establish the equivalence of these integrals, thus giving the reader insight into what integration is all about.

For topics beyond calculus, the concept of a metric space is crucial. Chapter 6 describes topology in metric spaces as well as the notion of compactness, especially with regard to the Heine–Borel theorem.

The subject of metric spaces leads in a natural way to the calculus of functions in N-dimensional spaces with $N > 2$. Here derivatives of functions of N variables are developed, and the Darboux and Riemann integrals, as described in Chapter 5, are extended in Chapter 7 to N-dimensional space.

Infinite series is the subject of Chapter 8. After a review of the usual tests for convergence and divergence of series, the emphasis shifts to uniform convergence. The reader must master this concept in order to understand the underlying ideas of both power series and Fourier series. Although Fourier series are not included in this text, the reader should find it fairly easy reading once he or she masters uniform convergence. For those interested in studying computer science, not only Fourier series but also the application of Fourier series to wavelet theory is recommended. (See, e.g., *Ten Lectures on Wavelets* by Ingrid Daubechies.)

There are many important functions that are defined by integrals, the integration taken over a finite interval, a half-infinite integral, or one from $-\infty$ to $+\infty$. An example is the well-known Gamma function. In Chapter 9 we develop the necessary techniques for differentiation under the integral sign of such functions (the Leibniz rule). Although desirable, this chapter is optional, since the results are not used later in the text.

Chapter 10 treats the Riemann–Stieltjes integral. After an introduction to functions of bounded variation, we define the R-S integral and show how the usual integration-by-parts formula is a special case of this integral. The generality of the Riemann–Stieltjes integral is further illustrated by the fact that an infinite series can always be considered as a special case of a Riemann–Stieltjes integral.

A subject that is heavily used in both pure and applied mathematics is the Lagrange multiplier rule. In most cases this rule is stated without proof but with applications discussed. However, we establish the rule in Chapter 11 (Theorem 11.4) after developing the facts on the implicit function theorem needed for the proof.

In the twelfth, and last, chapter we discuss vector functions in $\mathbb{R}^N$. We prove the theorems of Green and Stokes and the divergence theorem, not in full generality but of sufficient scope for most applications. The ambitious reader can get a more *general* insight either by referring to the book *A First Course in Real Analysis* or the text *Principles of Mathematical Analysis* by Walter Rudin.

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