Pre	REACE	PAGE V
	CHAPTER I. THE GENERAL NUMBER.	·
A. '	THE SYSTEM OF NUMBERS.	
1.	Addition and Subtraction. Origin of numbers. Counting and measuring. Addition. Subtraction as reverse operation of	
2	addition	1
	into positive and negative.	2
3.	Negative number a mathematical conception like the imaginary number. Cases where the negative number has a physical	
4.	meaning, and cases where it has not Multiplication and Division. Multiplication as multiple addi- tion. Division as its reverse operation. Limitation of divi-	4
	sion	6
5.	The fraction as mathematical conception. Cases where it has a	
6.	physical meaning, and cases where it has not Involution and Evolution. Involution as multiple multiplica-	8
	tion. Evolution as its reverse operation. Negative expo-	
_	nents	9
7.	Multiple involution leads to no new operation	10
ð. 0	Irrational Numberg Limitation of avalution Fudless desired	10
9.	fraction Bationality of the irrational number	11
10.	Quadrature numbers. Multiple values of roots. Square root of	11
	tion by 90°	13
11.	Comparison of positive, negative and quadrature numbers. Reality of quadrature number Cases where it has a physical	10
	meaning, and cases where it has not	14
12.	General Numbers. Representation of the plane by the general	
12	number. Its relation to rectangular coordinates	16
10.	Number of such roots, and their relation	18
14.	The two reverse operations of involution	19
	. • xi	

		AGE
15.	Logarithmation. Relation between logarithm and exponent of	
	involution. Reduction to other base. Logarithm of negative	-
	quantity	20
16.	Quaternions. Vector calculus of space	22
17.	Space rotors and their relation. Super algebraic nature of space	99
ъ	A STREET OF THE CONTRACT NUMBER OF COMPLEX OUANTITY	24
	Determine and Deler Coordinates	95
10	Rectangular and Polar Coordinates	20
18.	Powers of j. Ordinary or real, and quadrature or infaginary	95
	number. Relations	23
19.	Conception of general number by point of plane in rectangular	
	coordinates; in polar coordinates. Relation between rect-	96
	angular and polar form	26
20.	Addition and Subtraction. Algebraic and geometrical addition	
	and subtraction. Combination and resolution by parallelo-	0.0
	gram law	28
21.	Denotations	30
22.	Sign of vector angle. Conjugate and associate numbers. Vec-	•
	tor analysis	30
23.	Instance of steam path of turbine	33
24.	Multiplication. Multiplication in rectangular coordinates	38
25.	Multiplication in polar coordinates. Vector and operator	38
26.	Physical meaning of result of algebraic operation. Representa-	
	tion of result	40
27.	Limitation of application of algebraic operations to physical	
	quantities, and of the graphical representation of the result.	
	Graphical representation of algebraic operations between	
_	current, voltage and impedance	40
28.	Representation of vectors and of operators	42
29.	Division. Division in rectangular coordinates	4 2
30.	Division in polar coordinates	43
31.	Involution and Evolution. Use of polar coordinates	44
32.	Multiple values of the result of evolution. Their location in the	
	plane of the general number. Polyphase and n phase systems	
	of numbers	45
33.	The <i>n</i> values of $\sqrt{1}$ and their relation	4 6
34.	Evolution in rectangular coordinates. Complexity of result	47
35.	Reduction of products and fractions of general numbers by polar	
	representation. Instance	4 8
36.	Exponential representations of general numbers. The different	
	forms of the general number	4 9
37.	Instance of use of exponential form in solution of differential	
	equation	50

38	Interview Resolution of the logenithm of a general	AGE
00.	number	51
	CHAPTER II. THE POTENTIAL SERIES AND EXPONENTIAL FUNCTION.	
A. (General.	
39.	The infinite series of powers of x	52
40.	Approximation by series	53
41.	Alternate and one-sided approximation	54
42.	Convergent and divergent series	55
43.	Range of convergency. Several series of different ranges for	
•	same expression	56
44 45.	Discussion of convergency in engineering applications Use of series for approximation of small terms. Instance of	57
	electric circuit	58
4 6.	Binomial theorem for development in series. Instance of in-	
	ductive circuit	59
47.	Neccessity of development in series. Instance of arc of hyperbola	60
48.	Instance of numerical calculation of log $(1+x)$	63
B. 1	Offerential Equations.	
49.	Their typical forms	64
50.	$\frac{dy}{dx} = y$. Solution by series, by method of indeterminate co-	
	efficients	65
51.	$\frac{dz}{dx} = az$. Solution by indeterminate coefficients	68
52.	Integration constant and terminal conditions	68
53.	Involution of solution. Exponential function	70
54 .	Instance of rise of field current in direct current shunt motor	72
55.	Evaluation of inductance, and numerical calculation	75
56.	Instance of condenser discharge through resistance	76
57.	Solution of $\frac{d^2y}{dx^2} = ay$ by indeterminate coefficients, by exponential	
	function	78
58.	Solution by trigonometric functions	81
59.	Relations between trigonometric functions and exponential func-	
	tions with imaginary exponent, and inversely	83
60.	Instance of condenser discharge through inductance. The two	
	integration constants and terminal conditions	84
61.	Effect of resistance on the discharge. The general differential	
	equation	86

	I	AGE
62.	Solution of the general differential equation by means of the	
	exponential function, by the method of indeterminate	
	coefficients	86
63.	Instance of condenser discharge through resistance and induc-	
	tance. Exponential solution and evaluation of constants	88
64.	Imaginary exponents of exponential functions. Reduction to	
• 11	trigonometric functions. The oscillating functions	91
65.	Explanation of tables of exponential functions)	92

CHAPTER III. TRIGONOMETRIC SERIES.

А. Т	FRIGONOMETRIC FUNCTIONS.	
66.	Definition of trigonometric functions on circle and right triangle	94
67.	Sign of functions in different quadrants	95
68.	Relations between sin, cos, tan and cot	97
69.	Negative, supplementary and complementary angles $\dots \dots \dots$	98
70.	Angles $(x \pm \pi)$ and $\left(x \pm \frac{\pi}{2}\right)$	100
71.	Relations between two angles, and between angle and double	
	angle	102
7 2.	Differentiation and integration of trigonometric functions.	
	Definite integrals	103
73.	The binomial relations	104
74.	Polyphase relations.	104
75.	Trigonometric formulas of the triangle	105
В. Л	Trigonometric Series.	
76.	Constant, transient and periodic phenomena. Univalent peri-	
	odic function represented by trigonometric series	106
77.	Alternating sine waves and distorted waves	107
78.	Evaluation of the Constants from Instantaneous Values. Cal-	
-	culation of constant term of series	108
79.	Calculation of cos-coefficients	110
80.	Calculation of sin-coefficients	113
81.	Instance of calculating 11th harmonic of generator wave	114
82.	Discussion. Instance of complete calculation of pulsating cur-	
	rent wave	116
83.	Alternating waves as symmetrical waves. Calculation of sym-	
	metrical wave	117
84.	Separation of odd and even harmonics and of constant term	120
85.	Separation of sine and cosine components	121
86.	Separation of wave into constant term and 4 component waves	122
87.	Discussion of calculation	123
88.	Mechanism of calculation	124

xiv

		PAGE
89.	Instance of resolution of the annual temperature curve	125
90.	Constants and equation of temperature wave	131
91.	Discussion of temperature wave	132
C. R	LEDUCTION OF TRIGONOMETRIC SERIES BY POLYPHASE RELATION	
92.	Method of separating certain classes of harmonics, and its	
	limitation	134
93.	Instance of separating the 3d and 9th harmonic of transformer	
	exciting current	136
D. (CALCULATION OF TRIGONOMETRIC SERIES FROM OTHER TRIGONO-	
	METRIC SERIES.	
94.	Instance of calculating current in long distance transmission line,	
	due to distorted voltage wave of generator. Line constants	139
95.	Circuit equations, and calculation of equation of current	141
96.	Effective value of current, and comparison with the current	
	produced by sine wave of voltage	143
97.	Voltage wave of reactance in circuit of this distorted current	145

CHAPTER IV. MAXIMA AND MINIMA.

98. Maxima and minima by curve plotting. Instance of magnetic	
permeability. Maximum power factor of induction motor as	
function of load 1	.47
99. Interpolation of maximum value in method of curve plotting. Error in case of unsymmetrical curve. Instance of efficiency	
of steam turbine nozzle. Discussion 1	49
100. Mathematical method. Maximum, minimum and inflexion	
point. Discussion 1	.52
101. Instance: Speed of impulse turbine wheel for maximum	
efficiency. Current in transformer for maximum efficiency. I	.54
102. Effect of intermediate variables. Instance: Maximum power	
in resistance shunting a constant resistance in a constant cur-	EE
rent circuit	100
105. Simplification of calculation by suppression of unnecessary terms,	157
104 Instance: Maximum non-inductive load on inductive transmis-	.01
sion line. Maximum current in line 1	158
105. Discussion of physical meaning of mathematical extremum.	
Instance 1	160
106. Instance: External reactance giving maximum output of alter-	
nator at constant external resistance and constant excitation.	
Discussion 1	161
107. Maximum efficiency of alternator on non-inductive load. Dis-	
cussion of physical limitations 1	162

I	AGE
108. Extrema with several independent variables. Method of math	163
ematical calculation, and geometrical meaning	
109. Resistance and reactance of load to give maximum output of transmission line at constant supply voltage	165
110 Dimension of a barried limitations	167
110. Discussion of physical multations	
tient. Instance: Maxima of current wave of alternator of	1 00
distorted voltage on transmission line	168
112. Graphical calculation of differential curve of empirical curve,	170
for determining extrema	170
113. Instance: Maximum permeability calculation	170
114. Grouping of battery cells for maximum power in constant resist-	
ance	171
115. Voltage of transformer to give maximum output at constant	
loss	173
116. Voltage of transformer, at constant output, to give maximum efficiency at full load, at half load,	174
117 Maximum value of charging current of condenser through	
inductive circuit (a) at low resistance; (b) at high resistance.	175
118. At what output is the efficiency of an induction generator a max-	
imum?	177
119. Discussion of physical limitations. Maximum efficiency at con-	
stant current output	178
120. METHOD OF LEAST SQUARES. Relation of number of observa-	
tions to number of constants. Discussion of errors of	
observation	179
121. Probability calculus and the minimum sum of squares of the	
errors.	181
122. The differential equations of the sum of least squares	182
123. Instance: Reduction of curve of power of induction motor	
running light, into the component losses. Discussion of	
results	182
123A. Diophantic equations	186

CHAPTER V. METHODS OF APPROXIMATION,

124.	Frequency of small quantities in electrical engineering problems.	
	Instances. Approximation by dropping terms of higher order.	187
125.	Multiplication of terms with small quantities	188
126.	Instance of calculation of power of direct current shunt motor .	189
127.	Small quantities in denominator of fractions	190
128.	Instance of calculation of induction motor current, as function	
	of slip	191

100 TT (1) 11 1 1 1 1 1	PAGE
129. Use of binomial series in approximations of powers and roots,	
and in numerical calculations	193
130. Instance of calculation of current in alternating circuit of low	
inductance. Instance of calculation of short circuit current	
of alternator as function of sneed	105
121 Use of exponential agrice and lowerithmic corrige in exponential	130
151. Use of exponential series and logarithmic series in approxima-	
tions	196
132. Approximations of trigonometric functions	198
133. McLaurin's and Taylor's series in approximations	198
134. Tabulation of various infinite series and of the approximations	
derived from them	100
125 Estimation of accuracy of approximation Application to	100
155. Estimation of accuracy of approximation. Application to	
short circuit current of alternator	200
136. Expressions which are approximated by $(1+s)$ and by $(1-s)$.	201
137. Mathematical instance of approximation	203
138. EQUATIONS OF THE TRANSMISSION LINE. Integration of the	
differential equations	204
120 Substitution of the terminal conditions	201
149. Substitution of the terminal conditions	200
140. The approximate equations of the transmission line	206
141. Numerical instance. Discussion of accuracy of approximation.	207
141A. Approximation by chain fraction	208
141B. Approximation by chain fraction	208c
· · · ·	

CHAPTER VI. EMPIRICAL CURVES.

A. GENERAL.	
142. Relation between empirical curves, empirical equations and	
rational equations	209
143. Physical nature of phenomenon. Points at zero and at infinity.	
Periodic or non-periodic. Constant terms. Change of curve	
law. Scale.	210
B. Non-Periodic Curves.	
144. Potential Series. Instance of core-loss curve	212
145. Rational and irrational use of potential series. Instance of fan	
motor torque. Limitations of potential series	214
146. PARABOLIC AND HYPERBOLIC CURVES. Various shapes of para-	
bolas and of hyperbolas	216
147. The characteristic of parabolic and hyperbolic curves. Its use	
and limitation by constant terms	223
148. The logarithmic characteristic. Its use and limitation	224
149. EXPONENTIAL AND LOGARITHMIC CURVES. The exponential	
function	227
150. Characteristics of the exponential curve, their use and limitation	
by constant term. Comparison of exponential curve and	
hyperbola,	228

xvii

	PAGE
151. Double exponential functions. Various shapes thereof	231
152. EVALUATION OF EMPIRICAL CURVES. General principles of	
investigation of empirical curves.	233
153. Instance: The volt-ampore characteristic of the tungsten lamp,	
reduced to parabola with exponent 0.6. Rationalized by	
reduction to radiation law	235
154 The volt-ampere characteristic of the magnetite arc, reduced	
to hyperbole with exponent 0.5	238
155 Charge of electric current with charge of circuit conditions.	
155. Change of electric current with change of chourt of the	241
reduced to double exponential function of time from 144 by	
156. Rational reduction of core-loss curve of paragraph 112, ~	944
parabola with exponent 1.0 for higher densities to	211
157. Reduction of magnetic characteristic, for higher densities, to	
hyperbolic curve. Instance of the investigation of a hys-	946
teresis curve of silicon steel	240
C. PERIODIC CURVES.	
158. Distortion of sine wave by harmonics	255
139. Inita and hith harmonic. Peak, multiple peak, nat top and	955
160 Combined affect of third and fifth harmonia	200
161 Even harmonics. Unequal shape and longth of half wayoo	203
Combined second and third harmonic	266
162. Effect of high harmonics	260
163. Ripples and nodes caused by higher harmonics. Incommen-	200
surable waves	271
	. 1

CHAPTER VII. NUMERICAL CALCULATIONS.

164. METHOD OF CALCULATION. Tabular form of calculation	275
165. Instance of transmission line regulation	277
166. EXACTNESS OF CALCULATION. Degrees of exactness: magnitude,	
approximate, exact	279
167. Number of decimals	281
168. INTELLIGIBILITY OF ENGINEERING DATA. Curve plotting for	
showing shape of function, and for record of numerical values	283
169. Scale of curves. Principles	286
170. Logarithmic and semi-logarithmic paper and its proper use	287
171. Completeness of record	290
171A. Engineering Reports	200
172. RELIABILITY OF NUMERCIAL CALCULATIONS. Necessity of relia- bility in engineering calculations	200
173. Methods of checking calculations. Curve plotting	290 200
174. Some frequent errors	293a 293b

APPENDIX A. NOTES ON THE THEORY OF FUNCTIONS.

A	١.	GEN	TERAL	FUNCTIONS.

175. Implicit analytic functi	on. Explicit analytic function.	Reverse	
function	· · · · · · · · · · · · · · · · · · ·	•••••	294

					PAGE
176.	Rational functio	n. Integer	function.	Approximations	by
	Taylor's Theore	m			$\dots 295$
177.	Abelian integrals	and Abelian	functions.	Logarithmic inte	gral
	and evnonential	functions			296
170	Trigonomotrio inte	anala and this		upstions Uwnerk	
110.	rigonometric inte	grais and tri	,.	inclions. Hypert	
	integrals and hy	perbolic func	tions		297
179.	Elliptic integrals	and elliptic fu	inctions. T	heir double periodi	icity 298
180.	Theta functions.	Hyperelliptic	e integrals a	and functions	300
181.	Elliptic functions	in the motion	of the pend	dulum and the sur	ging
	of synchronous	machines			301
182.	Instance of the ar	c of an ellips	is		301
ъ с		o or an ompo			
B. S	PECIAL FUNCTIONS	•			
1 83.	Infinite summatio	n series. Infi	nite product	t series	302
184.	Functions by inte	gration. Inst	tance of the	propagation funct	ions
	of electric wave	and impulse	S		303
185	Functions defined	by definite i	ntegrals		305
196	Instance of the m	mma functio	nograno ,	••••••	306
100.	Instance of the g	amma runcuo			300
C. I	Exponential, Tric	ONOMETRIC A	ND HYPERB	OLIC FUNCTIONS.	
187.	Functions of real	variables			306
188.	Functions of imag	zinary variabl	es		308
180	Functions of com	nley variable	3		308
100	Deletiona	PICA THIRDIC		· · · · · · · · · · · · · · · · · · ·	300
190.	netations	• • • • • • • • • • • •	• • • • • • • • • • •	• • • • • • • • • • • • • • • • • •	309

APPENDIX B. TABLES.

TABLE I.	Three decimal exponential functions	312
TABLE II.	Logarithms of exponential functions	
Expon	ential functions	313
Hyper	bolic functions	314
INDEX		315

xix